Answer sheet 1

Sheet 1

Task 5 Minimum: $(-0.3, -7.9)$, roots: 0.59, -1.19.

Task 6 Roots: $-\frac{\sqrt{79}}{10} - \frac{3}{10}$ and $+\frac{\sqrt{79}}{10} - \frac{3}{10}$, or -1.188 and 0.589 to 3d.p.

Task 7 Max: $(-5.3, 138.2)$ Min: $(-3.1, -60.2)$ Max: $(-0.4, 108.5)$ Min: $(1.6, -36.9)$.

Roots: $-6, -4, -2, 1, 2$.

Task 9 Asymptotes: $x = 2$, $x = -2$ Roots: $-0.714, 1.912, -2.199$ Max: $(1.505, 1.929)$ Min: $(2.497, 3.944)$.

Task 10 Asymptote: $x = 2$, Root: -0.839 Max: $(1, -1)$ Mins: $(0.382, -1.090)$, $(2.618, 10.090)$. Does not have asymptotes as x tends to $\pm\infty$.

Sheet 2

Problem 1 $2(x-1)(2x-15)$, $x = 1$, $0 \leq \alpha \leq 1$, $4y^2 - 2(\lambda+1)y + \alpha\lambda$ (after removing common factor of a^2). Min at $x = 1$, $y = 1$.

Problem 2 $\alpha = 0.5$: $\theta = \pi$. $\alpha = 1$: $\theta = 2\pi$. Table entries: $1.766, 2.310, 3.142, 3.973$.

Problem 3 $c = 0.404$.

Sheet 3

Problem 4 (i) $(0, 4)$ (ii) $(-2, 0)$, (i) $(0, 4)$ (ii) $(2, 0)$, slopes 2 and -2.

Problem 5 AB: $m = \frac{13}{5}$, $y = \frac{13}{5}x + \frac{4}{5}$. AC: $m = \frac{3}{5}$, $y = \frac{3}{5}x - \frac{26}{5}$. BC: $x = 2$. Area: 25.

Problem 6 The lines are parallel. $c = \sqrt{13}$, $(0.832, 0.555)$.

Problem 7 $P = \sqrt{\frac{41}{2}}$, $x = 0.331$, $y = 0.883$.

Problem 8 $(1.75, 0.5)$, $P = 6.25$.

Sheet 4

Problem 9 $X = 175$, $Y = 50$, $P = 6250$.

Problem 10 First row: $3, 2, 3600$. Second row: $2, 8, 3600$. $x = 1080$, $y = 180$, $P = 8860$.

Problem 11 $400x + 300y = c$, $x + y \leq 20,000$, $x \geq 4000$, $y \geq 5000$, $x + y \geq 10000$, Answer: $x = 4000$, $7 = 6000$, $C = 3400000$.

Problem 12 Adults: most 22, least 9. Kids: most 33, least 15. Max profit 20.50.

Sheet 5

Problem 14 $ae - bd \neq 0$, a) $\frac{7}{6}, -\frac{1}{9}$ b) $0, -2$ c) $\frac{7}{5}, \frac{4}{5}$.

Problem 15 $\begin{pmatrix} 2 & 3 \\ 4 & -3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$, $\begin{pmatrix} 1 & -1 \\ 2 & -3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 6 \end{pmatrix}$, $\begin{pmatrix} 3 & -4 \\ -1 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$.

Problem 17 a) $-\frac{17}{5}, -\frac{13}{5}$ b) $\frac{5}{17}, \frac{13}{17}$ c) $\frac{10}{9}, \frac{-16}{9}$ d) No solutions.

Sheet 6

Problem 18 $\begin{pmatrix} 2 \\ -17 \\ 7 \end{pmatrix}$.

Problem 19 $\begin{pmatrix} 2 \\ -1 \\ -3 \end{pmatrix}$, $\begin{pmatrix} 38 \\ -1 \\ -10 \\ 11 \end{pmatrix}$, $\begin{pmatrix} \frac{13}{9} \\ \frac{1}{9} \\ \frac{1}{3} \\ \frac{1}{3} \end{pmatrix}$.

Problem 20 $\begin{pmatrix} 5 \\ 1 \end{pmatrix}$, $\begin{pmatrix} 1 \\ 11 \end{pmatrix}$, $\begin{pmatrix} 14 \\ 10 \end{pmatrix}$,

Problem 21 11, 2, 3, -5, 0. Final product impossible.

Problem 22 $\begin{pmatrix} -4 & 8 \\ -6 & 14 \end{pmatrix}$, $\begin{pmatrix} 12 & 12 \\ 2 & 8 \end{pmatrix}$, $\begin{pmatrix} 3 & 7 \\ 2 & 1 \end{pmatrix}$, $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $\begin{pmatrix} -4 & -4 \\ 12 & 14 \end{pmatrix}$.
$AB \neq BA$ in general.

Problem 24 a) $\begin{pmatrix} 1 & -1 & 3 & 0 \\ 2 & 2 & 4 & 0 \\ -2 & 0 & 2 & 2 \end{pmatrix}$, b) $\begin{pmatrix} -1 & 3 \\ -2 & 6 \end{pmatrix}$, c) impossible, d) $\begin{pmatrix} 2 & 1 & 3 \\ 2 & -2 & 0 \end{pmatrix}$, e) impossible.

Problem 25 a) $\begin{pmatrix} 5 & -7 \\ -2 & 3 \end{pmatrix}$, b) $\begin{pmatrix} 5 & -3 \\ -3 & 2 \end{pmatrix}$, c) $\begin{pmatrix} \frac{7}{2} & -\frac{3}{2} \\ -2 & 1 \end{pmatrix}$, d) impossible, e) $\begin{pmatrix} \frac{3}{4} & -\frac{1}{4} & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{4} & -\frac{1}{4} & \frac{1}{2} \end{pmatrix}$,
f) $\begin{pmatrix} 1 & \frac{1}{2} & \frac{1}{2} \\ 1 & 1 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$, g) $\begin{pmatrix} -2 & \frac{2}{3} & \frac{7}{6} & \frac{1}{2} \\ -1 & -\frac{1}{3} & \frac{7}{6} & \frac{1}{2} \\ 1 & 0 & -\frac{1}{2} & -\frac{1}{2} \\ 1 & 0 & -1 & 0 \end{pmatrix}$.

Sheet 7

Problem 26 ± 1.895, 0.

Problem 27 $\frac{1}{4}$, $\frac{3}{4}$, for example $x = \frac{16x^2+3}{16}$.

Problem 28 $a = 1.90$, $x_2 = 1.82$, $x_3 = 1.94...$

Problem 29 $x_1 = 0.8$, $x_2 = 0.8275$, $x_3 = 0.8723$, $x_4 = 0.9483$, $x_5 = 1.0868$. Will not converge to a root. At 0.2 iteration converges to 0.75 not 0.25.

Problem 30 a) ± 1.237 $\left(x = \dfrac{3x + 2\sin 2x}{4}\right)$, 0 $\left(x = \dfrac{4x - 2\sin 2x}{3}\right)$, b) 0.7236 $\left(x = \dfrac{10x - 2}{10x}\right)$, 0.2764 $\left(x = \dfrac{10x^2 + 2}{10}\right)$, c) 0.7391 $(x = \cos x)$, d) -2.9122 $\left(x = \dfrac{-2x^2 + 3x + 1}{x^2}\right)$, -0.2865 $\left(x = \dfrac{x^3 + 2x^2 - 1}{3}\right)$, 1.1987 $\left(x = \dfrac{x^3 + 2x^2 - 9x - 1}{-6}\right)$, e) 1.247 $\left(x = 1 + \dfrac{x^2 \cos x}{2}\right)$.

Sheet 8

Problem 31 a) $3x^2 + 6x + 1$, b) $4x^3 + 2x$, c) $\dfrac{-2}{(x-1)^2}$, d) $2x\cos(x^2)$, e) $2x\cos(x) - x^2\sin(x)$, f) $\ln(x^2+1) + \dfrac{2x^2}{x^2+1}$, g) $\dfrac{1}{\cos(x)^2}$.

Problem 32 a) $y = 15x - 9$, b) $y = 10x - 7$, c) $y = -\dfrac{x}{2} + \dfrac{3}{2}$, d) $y = x$, e) $y = \ln(2)x$, f) $y = (e+1)x - 2$.

Problem 33 1, 1.114729, 1.114157, etc.

Problem 34 a) -4.278, b) 0, ± 0.9477, c) ± 0.7226, d) 1.753.