Worksheet 1

Curve sketching using the DERIVE package

This sheet introduces the package DERIVE for windows. This package is a useful mathematical tool which we will use throughout the course. In this first sheet you will learn how to use it to investigate functions of one variable; plotting the graph of $f(x)$, finding roots of $f(x) = 0$, estimating where local maxima and minima occur and examining asymptotes.

Task 1 Entering DERIVE

Login to the system using your user id and password.

From the **Start** menu (bottom left corner) pull down the **Programs** menu and then the **D: Maths and Statistics** menu. Select the option **Derive**. Some PCs may not have this option – all laboratory computers should have it. If you don't have it as an option inform the person running the laboratory. You may have to use a different computer.

Task 2 Inputing and plotting a function $y = f(x)$.

There should be a long white box at the bottom of the DERIVE window. This is the box where you "Author" an "Expression". If it is not there then pull down the menu for **Author** and select **Expression…**.

In the box at the bottom type $10x\hat{\ }2 + 6x - 7$ and then press return. Note the '^' symbol is got by pressing [Shift] 6.

You should now see something like

$\#1 : 10{\cdot}x^2 + 6{\cdot}x - 7$

highlighted. This means that the package has entered the function as expression 1, denoted by $\#1$.

Common to most window packages, there is usually more than one way to perform any one action. For example, to bring up the author box one could also have either clicked on the button

or pressed the F2 key. Try using both of these options. From now on, these notes will generally only tell you one route to achieving a result, but be aware that the route given may not be the fastest and experiment with the different menus and shortcut buttons. Try clicking on the different mouse buttons when you are in different windows. N.B., it is advisable to click on the lower

button at the top right so that you can easily keep track of the different plot and algebra windows you will create.

Now plot the function as follows.

Pull down the **Window** menu and select **New 2d-Plot Window**. You should see a new display with a set of axes and a new set of menus and short cut buttons.

Pull down the **Insert** menu and select **Plot**.

You should see the graph of the curve $10x^2 + 6x - 7$, or at least part of it.

Task 3 Viewing a curve

Note that the graph produced in task 2 is displayed over a range which is preset by DERIVE. We may want to chose a different scale in the x or y-direction or both and may want to chose a different position of the origin. The current scale and the position of the origin are listed at the foot of the screen.

Use the **Set** menu and select either **Plot region...** or **Plot range...** to alter the values of x and y shown. For example use **Set** → **Plot region...** and change the vertical length to 20 and the vertical intervals to 10.

Try experimenting with the shortcut buttons to see what effect they have on the scales. What happens when you click on the following buttons?
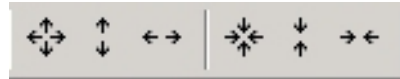
Task 4 Moving the cross

Move the mouse to position the cursor at some point on the screen and click. You should see the position of the cross move to the point you have clicked. Notice that the coordinates of the cross are given at the bottom of the screen.

Can you find another way of moving the cross using the pull-down menus?

What happens when you click on the following buttons?

Task 5 Locating points of interest on the curve

Move the cross to the point on the curve where the function crosses the y-axis. Check that this occurs when $y = -7$. Is this what you expect? Zoom in on this point for greater accuracy.

Now move the cross to the local minimum. By zooming in estimate the coordinates of this point to one decimal place. Write your result below.

Coordinates of the minimum are (,   ) to one decimal place.

Now look at the two points where the curve crosses the x-axis. These give the values of the roots of the quadratic, that is the solutions of $10x^2 + 6x - 7 = 0$. Move the cross to each point in turn and estimate the value of the roots to two decimal places. Write your answer below

The roots of $10x^2 + 6x - 7$ are $x_1 = $ and $x_2 = $ to two decimal places.

Check your answers using the formula for solutions of a quadratic equation, that is,

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Task 6 Using DERIVE to calculate the roots

Return to the first screen containing the algebraic expression $\#1{:}10x^2 + 6x - 7$ by pulling down the **Window** menu and selecting the **1 Algebra 1** option at the bottom, or by clicking on the window if you can see it.

Pull down the **Solve** menu and select the **Expression...** option. A window will pop up. Select **Algebraically** under solution method and click on the **Solve** button.

The pop-up window will go. A new expression listing the roots should appear in the main window. Note that in this case you did not tell DERIVE that you wanted to solve expression $\#1$ equal to zero. It assumes, by default, that if you do not give it an explicit equation it should set the expression equal to zero.

The expressions for the roots are exact. They are given as $x = \ldots \ \lor \ x = \ldots$. The symbol $\lor$ means "or".

The expressions for the roots to $10x^2 + 6x - 7 = 0$ obtained by DERIVE are $x_1 = $ and $x_2 = $.

Find a numerical value to three decimal places for x_1 by highlighting the expression for x_1 (don't include the x itself, this is done by clicking on the expression until the required part is

highlighted) and then selecting **Approximate** from the **Simplify** menu. A box should appear asking for the required number of digits of precision. Change this to the required value and click on **OK**. Use **Simplify** $\to$ **Basic** to evaluate this expression. In a similar way find a numerical value for x_2. (You could also try the **Approximate** button on the pop-up window.)

The numerical values for the roots to $10x^2 + 6x - 7 = 0$ obtained by DERIVE are $x_1 =$
and $x_2 =$ to three decimal places.

Compare the values that you obtained algebraically with those you obtained from the graph.

Now tidy everything up ready for a new task. Highlight expressions you wish to delete with the mouse button and select **Delete** from the **Edit** menu, or find the **Delete** button.

Task 7 Studying a more complicated curve

Enter a new expression by entering in the author box $x\hat{}5 + 9x\hat{}4 + 10x\hat{}3 - 60x\hat{}2 - 56x + 96$. (see Task 2 if you have forgotten how.) You should see

$\#1 : x^5 + 9{\cdot}x^4 + 10{\cdot}x^3 - 60{\cdot}x^2 - 56{\cdot}x + 96.$

This is a polynomial of degree 5 (a quintic). Once again plot out the curve, as you did in **Task 3**. Make sure that when you enter the plot window you delete the graph of the quadratic function which we studied in tasks 2 to 6, before asking DERIVE to plot the new function.

Play around with the scales and position of the origin so that you get a good view of the curve. A rteasonable range to use is x between -7 and $+3$, and y between -250 and $+250$.

Satisfy yourself that the curve has two local maxima, two local minima and that the function has five real roots (i.e., the curve crosses the x-axis five times). Check that the curve crosses the y-axis at 96.

Estimate the position of each turning point to one decimal place by moving the cross and zooming in for greater accuracy. List your answers below.

Coordinates of first maximum are (,   ).

Coordinates of first minimum are (,   ).

Coordinates of second maximum are (,   ).

Coordinates of second minimum are (,   ).

Similarly estimate where each root occurs and write down the values below.

$x_1 =$, $x_2 =$, $x_3 =$,
$x_4 =$, $x_5 =$.

Task 8 Simplifying the function

Return to the algebra window.

Make sure that the quintic expression is highlighted and select **Factor** from the **Simplify** menu. A box giving you five options, Trivial, SquareFree, Rational, Radical and Complex, should appear. Select rational and then click on the **Factor** button.

A second expression should appear with the quintic written in factorized form, a simpler form than the original form. You should be able to read off the exact values for all the roots trivially. Check these values with those estimated from the graph of the function.

Task 9 Asymptotes of a function.

Again tidy everything up, removing any expressions and graphs you have generated. Then enter the function.

$(x^3 + x^2 - 4x - 3)/(x^2 - 4)$

If you cannot remember how to do this, look back at tasks 2 and 7. This curve has two vertical asymptotes (are you able to see what they are from the form of the function?)

Plot out the function as before.

Experiment with the scale to get a good view of the curve. A good scale to use is x between -4 and 4, and y between -10 and 10. You should see that the curve is divided into three

separate pieces. It crosses the x-axis at three distinct points and the y-axis at one point. Use the cross to find the positions of the two vertical asymptotes.

Position of the first asymptotes is $x =$.

Position of the second asymptote is $x =$.

Check that the curve crosses the y-axis at $y = 0.75$. Use the cursor to find the three positions where the curve crosses the x-axis. Hence list the three roots of $(x^3 + x^2 - 4x - 3)/(x^2 - 4) = 0$

First root is $x =$.

Second root is $x =$.

Third root is $x =$.

Return to the algebra window and use solve to find the roots of the function algebraically. (Select **Algebraically** from the **Solve** $\rightarrow$ **Expression** menu). DERIVE should give you three complicated looking expressions. Numerical approximations to these expressions can be obtained by highlighting and selecting **Approximate** from the **Simplify** menu. List the results below.

DERIVE calculates the roots to be $x_1 =$, $x_2 =$ and $x_3 =$.

Compare these answers with the ones you obtained graphically. What happens if you use **Numerically** instead of **Algebraically** from the **Solve** $\rightarrow$ **Expression** menu on the original expression?

You should see that the curve has a local maximum and a local minimum. Find the coordinates of these points to one decimal place.

The position of the local maximum is (,   ).

The position of the local minimum is (,   ).

Looking at the curve you should notice that it approaches a straight line as $x \rightarrow \pm\infty$, that is, it has another asymptote in addition to the two vertical ones discussed above. Looking at the function for large and small x, satisfy yourself that it approaches the line $y = x + 1$. To get a better idea of this, return to the algebra window and enter the new expression

$x + 1$

Then return to the plot window and select **Insert** $\rightarrow$ **Plot**. You should now see both your original curve and the line $y = x + 1$. You should see that the curve really does approach $y = x + 1$ as $x \rightarrow \pm\infty$.

Task 10

Investigate the behaviour of the function $(x^3 - 2x^2 + 2)/(x - 2)$ using DERIVE. Check out any local maxima and minima, real roots and asymptotes. How does the function behave as $x \rightarrow \pm\infty$.