Mathematical Modelling of Biological Processes: The Heartbeat

3rd year project 2010/11

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Introduction

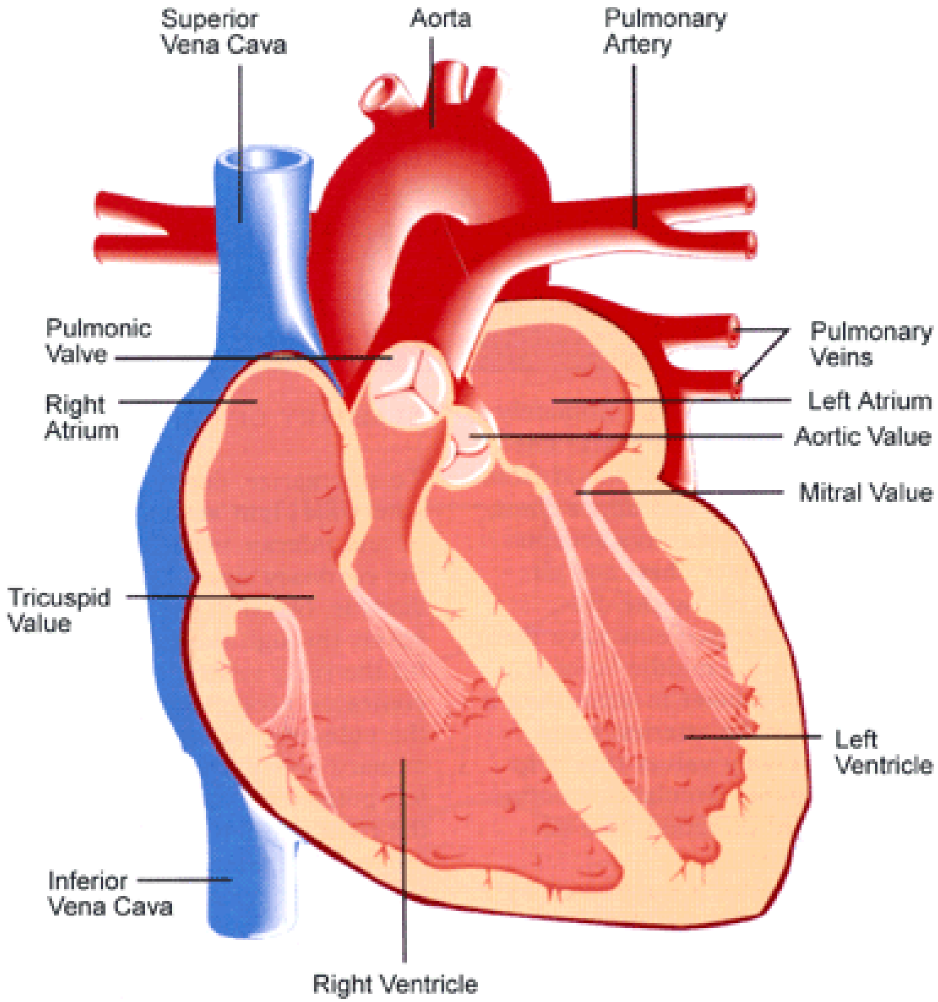
- The main idea of this project is to study a system of non-linear differential equations which were proposed by Prof. E.C. Zeeman in the seventies as a basic description of how the heart works.
- A review of the motivation for these equations and some suggested exercises can be found in the book

  "Differential Equations and Mathematical Biology" by D.S. Jones, M.J. Plank and B.D. Sleeman

  and also *at the web page:*

  http://www.geom.uiuc.edu/education/UMTYMP/CalcIII/1994/StudentLabs/Heartbeat/lab.html
- In fact if you "google" E.C. Zeeman you will find a lot of pages with information about his contributions to science and biography.

The Heart



http://www.niaaa.nih.gov/Resources/GraphicsGallery/CardiovascularSystem/PublishingImages/271f2.gif

The model

- In order to model the heartbeat we have to be able to incorporate the main properties of the heart in a mathematical model. The properties that are considered as fundamental in the model you will study are:

- The existence of an equilibrium state (fixed point) corresponding to the diastole (relaxed state of the heart)
- There must be a threshold for triggering the process whereby the heart contracts from a diastole to a systole (fully contracted state and another equilibrium state)
- The model should quickly return to the original equilibrium state after the systole.

E.C. Zeeman's Heart Equations

- Based on the three properties that we saw before, Prof. E.C. Zeeman argued that the simplest system of equations that could describe the heartbeat would be of the form:

$$\varepsilon \frac{dx}{dt} = -(x^3 - Tx + b),$$

$$\frac{db}{dt} = x - x_0.$$

- T>0 and ε are constants which characterize the heart. T represents a tension

- The variable x represents the length of a muscle fibre in the heart

- The variable b represents an electrical control variable that triggers the electro-chemical wave leading to the heart's contraction

Equation's analysis

- The equations in the previous page can now be studied by using the methods of dynamical systems:
- The system can be linearized about its fixed point and the nature of the linear system's fixed point can be studied (e.g. saddle point, stable node, focus…)
- The phase diagram of the linear equation can be drawn and studied. The linear equations can be solved.
- For the non-linear equation it is harder to solve the equations and draw the phase diagrams, but this can be done using some mathematical software.
- Once all this analysis has been done we can then go on to analyse what happens to the heart when the values of T or ε are changed.