The theory of special relativity

3rd year project 2007/08

Supervisor: Olalla Castro Alvaredo

- The idea of this project is to familiarise yourselfs with the main ideas involved in the theory of special relativity.
- Of course, the scientist Albert Einstein was someone very interesting and the probably the most famous physicist in history, so the first choice you will have to make is how much of your project you want to dedicate to Einstein's biography (including his scientific contributions).
- In any case this is a project in maths, so you will have to have some maths in the project!

- In order to help you a bit with this I have written a document which is based on several books. The document contains quite a bit of maths and several small problems you could try to do.
- Working out those problems in detail will already count as a contribution to your project.
- Apart from the document I will give you there are many books (check our library!) and internet sites which deal with special relativity so you should try to find other sources. In particular, in the web you will find many pages with simple introductions to the topic (easier than in my notes).

The main idea…

- What is special relativity? It is a physical theory which describes how physical quantities (such as energy, position, time …) and physical laws change when objects move at velocities close to the speed of light w.r.t. the observer.
- Why does one need a new theory for velocities close to the speed of light? Because physical experiments show that the speed of light is the maximum speed that can be reached by anything in our universe. Once we admit that there is a maximum velocity, the usual physics laws fail. (See example later)

- The theory of special relativity is based on two main principles:
- The relativity principle of Galileo (1564-1642!): all movement is relative.
- The universality of the speed of light (established by the Michelson-Morley experiment 1887): the speed of light is a universal constant, which is independent of the relative velocity between the source emiting light and the observer. In adition, no material object can travel faster than light.

- Galilean relativity: the train example (courtesy of Stephen Hawking).
- Two people riding on a train from New York to San Francisco play a game of ping-pong in the sport compartment of the train. Lets say, the train moves at 100 km per hour (= 27.8 m/s) and the two players hit the ball at a speed of two meters per second. In the reference frame of the players, the ball moves back and forth at this particular speed. For a stationary observer standing beside the railroad, however, things look quite different. In his reference frame the ball moves at 29.8 m/s when it is played forward in the direction where the train is heading, while it moves at 25.8 m/s in the same direction when it is played backwards. Thus he doesn't see the ball moving backward at all, but always moving towards San Francisco. For an observer in outer space, things look again totally different because of the Earth's rotation, which is opposite to the train's movement; therefore the outer space observer always sees the ball moving East.
- From Galileo's principle of relativity follows a particular law of composition of velocities! Also there is the assumption that the time of the observer and of the players is the same.
- I found this example at: http://www.thebigview.com/spacetime/relativity.html

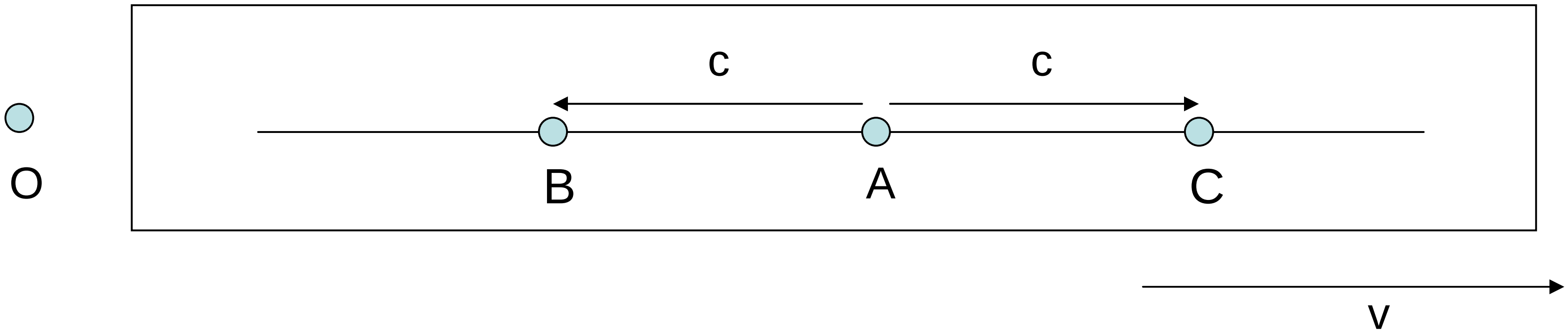
- Once we accept that there is no velocity larger than the speed of light, then the law of composition of velocity of Galileo starts to fail.
- We just need to imagine that the train of the previous story travels at the speed of light and that there is still this game of ping-pong going on. If we call the speed of light c then the person standing besides the railroad sees the ball going forward at speed (c+2) m/s and backwards at speed (c-2) m/s. But we just said that no speed can be bigger than c, so the speed c+2 is not allowed by the theory of relativity.
- This implies that there must be a new law of composition of velocities.

- Part of your project should be the study of the new laws of composition of velocities and position and time that follow from the theory of relativity (in particular explain how the phenomena of contraction of time and dilatation of space follow from this).
- One of the most famous consequences of relativity is the famous law $E=mc^2$ which expresses the fact that for particles moving at speed close to the speed of light the mass is just a certain form of energy (rest energy)
- A derivation of this equation would also be a nice contribution to the project. You could also provide examples from physics of how this equation works.

- A consequence of the fact that there is a limit to the speed that can be reached by anything is that light moves at the same speed (the maximum speed) no matter from which reference system we look at it.
- This constradicts again our standard ideas about composition of velocities, which are based on Galileo’s work.
- A further consequence of this is that things that happen at the same time for some observer can happen at different times w.r.t. a different observer (so there is not an absolute time!)
- Let us see an example of this!

An observer at the point A sends two light signals in oposite directions. He sends them simulateneously. Since the two signals move at the same speed and the two points B, C are at the same distance from A, the two signals will arrive at B and C simultaneously.

Let us now analize what the observer O sees. With respect to him the three points A, B and C are moving away at some velocity v while he is staying in the same place all the time.



At a given time A sends the two signals. With respect to O they also travel at speed c (since the speed of light is universal) but whereas the point B is moving away from O and therefore in the oposite direction as the direction of propagation of the signal going from A to B, the point C is moving away from O and in the same direction as the signal going from A to C. Therefore the signal must arrive at B earlier than it arrives at C (they do not arrive at the same time!)

- In special relativity we look at events that happen at points (t,x,y,z) characterized by 3 space coordinates and one time coordinate. That is a 4 dimensional space (usually called Mikovsky space).
- Similarly all physical quantities that are normally associated with vectors (like position, momentum, velocity etc) become 4-vectors in this new 4-dimensional space.
- If you want to go into more detail about the maths of relativity, then you will have to look at the properties of these objects!
- There are also some consequences of relativity that seem counter-intuitive: twin paradox, Doppler effect etc (have a look!)