COURSEWORK 1

To be handed in by 4:00 on Monday 14th of December

1. Find and classify the stationary points (maxima, minima and saddle points) of the following functions:
$$f(x,y) = y^3 + yx^2 - 6x^2 - 6y^2 + 9y,$$
13 points

2. Use the method of Lagrange multipliers to find the points on the sphere
$$(x-2)^2 + (y+1)^2 + (z-1)^2 = 4,$$
which are at maximum and minimum distance from the point $(0,0,1)$. In each case, compute also the value of the Lagrange multiplier. 20 points

3. By employing the change of variables
$$u = x - y \qquad \text{and} \qquad v = x + y,$$
evaluate the double integral
$$I = \int\int_R e^{\frac{x-y}{x+y}} \, dxdy,$$
where the integration region R is the triangle with vertices $(0,0)$, $(0,1)$ and $(1,0)$. Solve the problem in five steps:

   i) Compute the Jacobian of the variable transformation.

   ii) Write down the integrand $e^{\frac{x-y}{x+y}} dxdy$ in terms of the new variables u and v.

   iii) Sketch the integration region in the xy- and uv-planes.

   iv) From your sketch, determine the integration limits in terms of the new variables u and v.

   v) Compute the integral (choose the order of integration in u and v that makes the integral easiest to compute).

   20 points

4. By changing the order of integration, evaluate the double integral,
$$I = \int_{x=1}^{x=3} \int_{y=0}^{y=\ln x} x dy dx$$
Produce a rough sketch of the integration region. 12 points

5. Employ cylindrical coordinates to obtain the volume of the region bounded by the ellipsoid:
$$x^2 + y^2 + \frac{(z-1)^2}{4} = 1,$$
and the planes $z = -1/2$ and $z = 1/2$. 17 points

6. (a) Determine the functions $u_1(x)$ and $u_2(x)$ such that $y = c_1 u_1(x) + c_2 u_2(x)$ is the general solution of the homogeneous second-order differential equation
$$y'' - 9y = 0,$$
where c_1, c_2 are arbitrary constants. 5 points

   (b) Use the method of variation of parameters to find a particular solution of the inhomogeneous second-order differential equation
$$y'' - 9y = \frac{6}{\sinh(3x/2)}.$$
Hence determine the general solution of this inhomogeneous equation. 13 points