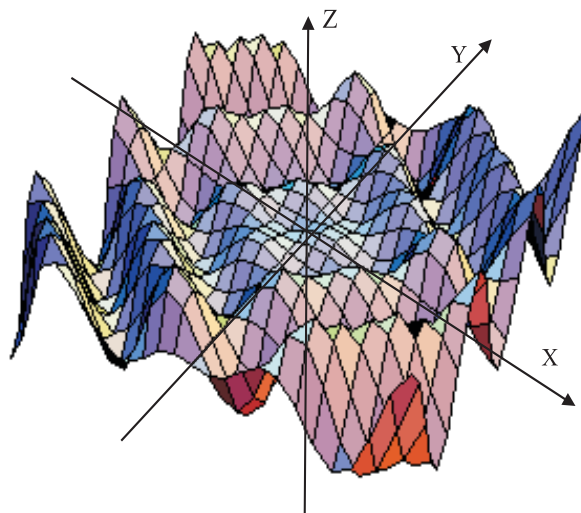


NOTES FOR MA2601 CALCULUS
ACADEMIC YEAR 2009/10

<http://www.staff.city.ac.uk/o.castro-alvaredo/teaching/calculus.html>



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Surgery hours: Wed. from 11:00 to 12:30 and from 15:00 to 16:30

Reading List

Some suitable textbooks for this module are:

1. Calculus: A complete course, by R.A. Adams (Addison Wesley).
2. Calculus, by H. Anton (Wiley).
3. Calculus: One and several variables, by S.N. Salas and E. Hille (Wiley).
4. University Calculus, by J. Hass, M.D. Weir and G.B. Thomas (Addison Wesley).

All the books above are of a similar level and content. Any of them will be useful for the course and will cover all or the majority of the material we will see in the course.

Dr Olalla Castro Alvaredo will not follow any of the books above in particular, but will take parts of different books as inspiration for her lectures.

Dr Olalla Castro Alvaredo will also provide her own notes for the course, which are based upon some of the books above. This notes, together with the notes you should take during the lectures should be sufficient for the course.

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