Sheet 2: partial derivatives

1. Calculate the 1st order partial derivatives of:

$$(a)\ f(x,y)=e^{x-y}-e^{y-x}, \qquad (b)\ g(x,y,z)=z\sin(x-y),$$
$$(c)\ \rho(\theta,\phi)=e^{\theta+\phi}\cos(\theta-\phi), \qquad (d)\ r(x,y)=\frac{x\sin(y)}{y\cos(x)}.$$
$$(e)\ h(x,y)=x^2y+y^3+x^3+y^2x, \quad (f)\ k(s,u)=s^u u^s.$$

2. Find f_x and f_y of the following functions

$$f(x,y)=x^2y, \qquad g(x,y)=\ln(x^2y),$$

by explicitly using the definitions

$$\begin{aligned} f_x &= \lim_{h\to 0}\frac{f(x+h,y)-f(x,y)}{h}, \\ f_y &= \lim_{h\to 0}\frac{f(x,y+h)-f(x,y)}{h}, \end{aligned}$$

and computing the corresponding limit.

3. The surface $z=\sqrt{4-x^2-y^2}$ is a hemisphere of radius 2 centered at the origin. Find the equation of the tangent line to the curve of intersection of the hemisphere with the plane $x=1$ at the point $(1,1,\sqrt{2})$.

4. Calculate the 2nd order partial derivatives of the functions:

$$(a)\ f(x,y)=\ln\left(\frac{x}{x+y}\right), \quad (b)\ g(x,y)=x^y \quad \text{and} \quad (c)\ r(x,y)=xe^y+ye^x.$$

5. Let

$$f(x,y)=\begin{cases} \frac{xy(y^2-x^2)}{y^2+x^2} & \text{if} \quad (x,y)\neq(0,0) \\ 0 & \text{if} \quad (x,y)=(0,0) \end{cases}.$$

Show that $f_{xy}(0,0)\neq f_{yx}(0,0)$.