Sheet 5: stationary points and Lagrange multipliers

1. Find and classify the extremes of the function
$$f(x,y) = x^2 + 2y^2 - 4x + 4y.$$

2. Find and classify the extremes of the function
$$f(x,y) = e^{2x+3y}(8x^2 - 6xy + 3y^2).$$

3. Find the minimum value of
$$f(x,y) = x + 8y + \frac{1}{xy},$$
in the first quadrant, e.g. $x > 0$ and $y > 0$.

4. Use the method of Lagrange multipliers to maximize x^3y^5 subject to the constraint $x + y = 8$.

5. Find the maximum and minimum values of the function $f(x,y,z) = x + y - z$ over the sphere $x^2 + y^2 + z^2 = 1$.

6. Minimize $f(x,y) = x^2 + y^2$ subject to the constraint $g(x,y) = x^2y - 16 = 0$.

7. Using the method of Lagrange multipliers, find the shortest distance from the point $(0,0,1)$ to the curve $y^2 + x^2 + 4xy - 4 = 0$ which lies in the xy-plane ($z = 0$).