Sheet 7: the method of variation of parameters

1. (a) Determine the functions $u_1(x)$ and $u_2(x)$ such that $y = c_1u_1(x) + c_2u_2(x)$ is the general solution of the homogeneous second-order differential equation

$$y'' - y = 0,$$

where c_1, c_2 are arbitrary constants. Show that the Wronskian of the functions u_1, u_2 is nowhere zero.
(b) Use the method of variation of constants to find a particular solution of the inhomogeneous second-order differential equation

$$y'' - y = \frac{1}{e^x + 1}.$$

Hence determine the general solution of this inhomogeneous equation.

2. (a) Determine the functions $u_1(x)$ and $u_2(x)$ such that $y = c_1u_1(x) + c_2u_2(x)$ is the general solution of the homogeneous second-order differential equation

$$y'' - 4y' + 5y = 0,$$

where c_1, c_2 are arbitrary constants. Show that the Wronskian of the functions u_1, u_2 is nowhere zero.
(b) Use the method of variation of constants to find a particular solution of the inhomogeneous second-order differential equation

$$y'' - 4y' + 5y = \frac{e^{2x}}{\sin x}.$$

Hence determine the general solution of this inhomogeneous equation.

3. (a) Determine the functions $u_1(x)$ and $u_2(x)$ such that $y = c_1u_1(x) + c_2u_2(x)$ is the general solution of the homogeneous second-order differential equation

$$y'' + 2y' + 2y = 0,$$

where c_1, c_2 are arbitrary constants. Show that the Wronskian of the functions u_1, u_2 is nowhere zero.
(b) Use the method of variation of constants to find a particular solution of the inhomogeneous second-order differential equation

$$y'' + 2y' + 2y = \frac{e^{-x}}{\cos^3 x}.$$

Hence determine the general solution of this inhomogeneous equation.