## Calculus: Coursework 1 Due in by 4:00pm, Thursday 20 November 2008

1. Use Lagrange multipliers to find the shortest distance from the origin to the hyperbola

$$
x^{2}+8 x y+7 y^{2}=225
$$

2. Sketch the region of integration in the $x-y$ plane for the following integral

$$
I_{1}=\int_{1}^{e} \int_{0}^{\ln x}\left[\frac{y}{x}+e^{-(2 y+1)} \cos \left(x e^{-(y+1)}\right)\right] d y d x
$$

Change the order of integration showing clearly what the new limits of integration should be, and hence evaluate the integral.
3. Find the Jacobian of the coordinate transformation

$$
x=u v \cos \phi, \quad y=u v \sin \phi, \quad z=\frac{1}{2}\left(u^{2}-v^{2}\right) .
$$

4. Use Laplace transforms to solve
(a) $y^{\prime \prime}+7 y^{\prime}+6 y=0$, with $y(0)=1$ and $y^{\prime}(0)=1$.
(b) $y^{\prime \prime}+6 y^{\prime}+13 y=1, y(0)=0$ and $y^{\prime}(0)=0$.
