Actuarial Science

CALCULUS: COURSEWORK 1 DUE IN BY 4:00PM, THURSDAY 20 NOVEMBER 2008

1. Use Lagrange multipliers to find the shortest distance from the origin to the hyperbola

$$x^2 + 8xy + 7y^2 = 225.$$

2. Sketch the region of integration in the x-y plane for the following integral

$$I_1 = \int_1^e \int_0^{\ln x} \left[\frac{y}{x} + e^{-(2y+1)} \cos\left(xe^{-(y+1)}\right) \right] \, dy \, dx.$$

Change the order of integration showing clearly what the new limits of integration should be, and hence evaluate the integral.

3. Find the Jacobian of the coordinate transformation

$$x = uv \cos \phi, \quad y = uv \sin \phi, \quad z = \frac{1}{2}(u^2 - v^2).$$

- 4. Use Laplace transforms to solve
 - (a) y'' + 7y' + 6y = 0, with y(0) = 1 and y'(0) = 1.
 - (b) y'' + 6y' + 13y = 1, y(0) = 0 and y'(0) = 0.