

Important: Read these instructions first

Question 1 involves the use of Derive. You should answer the questions in your answer book.

For questions 2, 3 and 4 you should download the Excel spreadsheet `CompMathProgTest10.xls` from the course web page at

www.staff.city.ac.uk/o.s.kerr/CompMaths/

You should enter answers to the questions in the boxes indicated on **Sheet1**. You can do your calculations on the other sheets, accessing them using the tabs towards the bottom of the sheets. You should also make sure that you define functions with the names indicated in the question, otherwise marks for correctly defined functions will be lost.

At the end of the test you should submit the spreadsheet to the Computational Mathematics module on CitySpace. **Remember to save your changes to the spreadsheet before submitting.** If you have difficulty submitting your Excel file to CitySpace then send it as an email attachment to `o.s.kerr@city.ac.uk`.

1. (a) Using Derive plot the function

$$f(x) = x^5 - x^4 - 14x^3 + 19x^2 + 7x + 5$$

Sketch this graph by hand in your solution book.

Identify the stationary points of the function, and write them down to 2 decimal places. [6 marks]

- (b) Plot the curves

$$g(x, y) = y^2 - x^2(x + 2) = 0$$

$$h(x, y) = x^2 + y^2 - 1 = 0$$

Sketch the curves in your answer book. Find all the roots to this system of equations to 3 decimal places. [6 marks]

Turn over . . .

2. You want to find the roots of the equation

$$f(x) = x^4 + x^2 + 4x - 3 = 0$$

This equation can be rearranged as

$$x = f_a(x) = \frac{3 - x^2 - x^4}{4}$$

Using Excel define the function to evaluate $f_a(x)$

```
Function fa(x)
....
End Function
```

Confirm that this iterative scheme $x_{n+1} = f_a(x_n)$ converges for an initial guess of $x_0 = 0.5$. Hence find the root near 0.5 to 4 decimal places. Enter this on Sheet1 of the Excel spreadsheet in the space indicated.

There is another root to $f(x) = 0$ near $x = -1.5$. Find a second rearrangement of $f(x) = 0$ in the form

$$x = f_b(x)$$

which converges near this root when used in an iterative scheme. Define the function **fb(x)** in Excel for this arrangement, and hence find the second root to 4 decimal places. Enter the root in the spreadsheet in the space indicated. [12 marks]

3. You want to find the roots of

$$f(x) = x^3 + 9 - 10 \cos(3x) = 0.$$

Using Newton's method derive an iterative scheme of the form

$$x_{n+1} = g(x_n)$$

Define the function **g(x)** in Excel for this iterative scheme, and use it to find the three roots of $f(x) = 0$ by iteration to 4 decimal places. Enter the roots in the spreadsheet in the spaces indicated. [10 marks]

Turn over ...

4. Using Excel find the products AB and BA of the matrices

$$A = \begin{pmatrix} 2 & 1 & 3 \\ 0 & -3 & -2 \\ 3 & -1 & 2 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 2 & 1 \\ 4 & 1 & -1 \\ 2 & -2 & -2 \end{pmatrix}$$

Find the inverses A^{-1} and B^{-1} of the matrices A and B , and enter into the spaces indicated. Determine whether the inverse of AB is $A^{-1}B^{-1}$ or $B^{-1}A^{-1}$, entering AB if the former or BA if the latter into the appropriate box in the spreadsheet. [8 marks]

5. You want to buy a second hand car for £4000 using money borrowed from the bank. You tell the bank that you do not want to pay more than £150 a month. If the bank charges 1.25% interest at the end of each month, find out the minimum number of months that will be required to repay the loan. Enter the Excel function and arguments used in the spreadsheet in the space indicated, and the number of months required to repay the loan.

Calculate the monthly repayments to the bank, given that all the repayments will be equal. Again enter the Excel function and arguments used in the spreadsheet in the space indicated, and the monthly repayments.

[8 marks]

Excel financial functions

The following financial functions are available in Excel:

=FV(r, n, PMT, PV, type)
=PV(r, n, PMT, FV, type)
=PMT(r, n, PV, FV, type)
=NPER(r, PMT, PV, FV, type)
=RATE(n, PMT, PV, FV, type, guess)

where

n = total number of periods;
r = interest rate per period;
PMT = value of each payment made during the transaction
PV = present value
FV = future value
type = 0 if the payments are at the end of each period.
guess = starting value for iterative RATE function, set =0

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Turn over . . .