

1 Introduction to Derive

Worksheet 1

This sheet introduces the package Derive for windows. This package is a useful mathematical tool which we will use throughout the course. In this first sheet you will learn how to use Derive to investigate functions of one variable: plotting the graph of $f(x)$; finding roots of $f(x) = 0$; estimating where local maxima and minima occur; examining asymptotes; differentiating and integrating $f(x)$;

Task 1 Entering Derive

Logon to the system using your id and password.

From the *Start* menu pull down the *Program* menu and then the *D.Maths and Statistics* menu. Select the option *Derive 5*.

(Please open Derive5 rather than any older version of Derive that may still be on the system)

Task 2 Inputting and plotting a function $y = f(x)$.

Derive refers to inputting an expression as **Authoring**. This can be done directly by left clicking with the mouse into the long thin **Authoring Window** at the bottom of the screen, (*just above all the symbols etc*) or alternatively by selecting **Author** from the top pull down menu and selecting **Expression**.

Enter $10x \wedge 2 + 6x - 7$ into the authoring window and strike the enter key.

Note the \wedge symbol is obtained by pressing [Shift] 6.


You should now see something like

#1 : $10x^2 + 6x - 7$

This means that the package has entered the function as expression 1, denoted by #1.

Common to most window packages, there is usually more than one way to perform any one action. From now on, these notes will generally only tell you one route to achieve a result, but be aware that the route given may not be the fastest and you should experiment with the different menus and shortcut buttons.

Now plot the function as follows:

- With the expression highlighted click on the plot 2-D button , you should see a new display with a set of axes and a new set of menus and short cut buttons.
- Click again on the 2-D plot button. (*6th button from the left*)

You should see the graph of the curve $10x^2 + 6x - 7$.

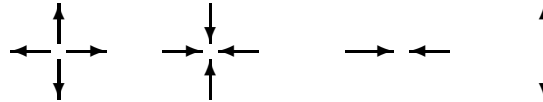
Task 3 Viewing a curve

Note that the graph produced in task 2 is displayed over a range which is preset by Derive. We may want to chose a different scale in the x or y -direction or both and may want to chose a

different position of the origin. The current scale and the position of the origin are listed at the foot of the screen.

Select **Set**, **Plot Range** from the top menu bar. Change the horizontal minimum range to -8 and the maximum to +8. Change the vertical minimum range to -20 and the maximum to +20.

Also try experimenting with the following **Zooming** shortcut buttons to see what effect they have on the scales.




Task 4 Moving the cross

Move the mouse to position the cursor at some point on the screen and click. You should see the position of the cross move to the point you have clicked. Notice that the coordinates of the cross are given at the bottom of the screen. The cross can also be moved using the arrow keys on the keyboard.

What happens when you click on:



Task 5 Locating points of interest on the curve - **curve tracing**

Place the cross near the curve and then click on the trace button  (9th button from the left.)

The cross becomes a small box located on the curve. This can be moved along the curve using the keyboard arrows \leftarrow and \rightarrow . The coordinates of the box are located in the bottom left hand corner of the plot.

Move the box cursor to the point on the curve where $x = 0$ and check that $y = -7$.

Now move the cursor to the local minimum to try and estimate its coordinates. By zooming in to improve the accuracy, estimate the coordinates of this point to one decimal place. Write your result below.

Coordinates of the minimum are (..... ,) to one decimal place.

Now look at the two points where the curve crosses the x -axis. (You may need to reset the plot range or use the zoom buttons to recapture your picture.) These points give the values of the roots of the quadratic, that is the solutions of $10x^2 + 6x - 7 = 0$. Move the tracing cursor to each point in turn and estimate the value of the roots to two decimal places. Write your answer below

The roots of $10x^2 + 6x - 7$ are $x_1 = \dots\dots\dots$ and $x_2 = \dots\dots\dots$ to two decimal places.

Check your answers using the formula for solutions of a quadratic equation, that is,

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Task 6 Using Derive to calculate the roots

Return to the first screen containing the algebraic expression #1: $10x^2 + 6x - 7$ by clicking the rightmost button on the menu bar or by pulling down the **Window** menu and selecting the **Algebra1** option.

Pull down the **Solve** menu and select the **Expression** option. (*Derive assumes you are solving the expression equal to zero*)

You should see a new options window.

- Select the method of solution to be algebraic (*this gives exact answers*) as opposed to the numerical option which give approximate answers.
- Select complex rather than real. This will solve the problem using complex numbers if necessary
- Click the **OK** button

You should now see the expression SOLVE ($10x^2 + 6x - 7, x$)

With this expression highlighted click **Simplify** followed by **Basic** from the top menu bar. The two exact roots should now be displayed, note that they are connected by the logical or symbol

∨

The expressions for the roots to $10x^2 + 6x - 7 = 0$ obtained by Derive are

$x_1 = \dots$ and $x_2 = \dots$

Find a numerical value to three decimal places for x_1 and x_2 by highlighting the results and then selecting **Approximate** from the **Simplify** menu. A box should appear asking for the accuracy you wish Derive to use to carry out the evaluation (*leave this set at 10*). Click on **Approximate**.

The numerical values for the roots to $10x^2 + 6x - 7 = 0$ obtained by Derive are

$x_1 = \dots$ and $x_2 = \dots$ to three decimal places.

Compare the values that you obtained above with those you obtained from the estimates you made tracing the graph.


Now tidy everything up ready for a new task. Highlight expressions you wish to delete with the mouse button and select **Delete** from the **Edit** menu or use the shortcut button χ (*eighth button from the left*).

Task 7 Studying a more complicated curve

Enter the new expression $x^5 + 9x^4 + 10x^3 - 60x^2 - 56x + 96$ into the authoring window and strike the enter key to obtain:

$$\#1 : x^5 + 9x^4 + 10x^3 - 60x^2 - 56x + 96.$$

This is a polynomial of degree 5 (a quintic). Plot the curve as follows:

- Enter the plot window and using the delete shortcut button χ delete the graph of the quadratic function studied in tasks 2 to 6.
- Plot the graph using the plot button 
- Set the plot range manually as before or just play around with the zoom buttons and the positioning buttons. (*ie origin at centre of screen button or cross at centre of screen button*)

Satisfy yourself that the curve has two local maxima, two local minima and that the function has five real roots (i.e. the curve crosses the x -axis five times). Check that the curve crosses the y -axis at 96.

Estimate the position of each turning point to one decimal place by tracing and zooming in for greater accuracy. List your answers below.

Coordinates of first maximum are (.....).

Coordinates of first minimum are (.....).

Coordinates of second maximum are (.....).

Coordinates of second minimum are (.....).

Similarly estimate where each root occurs and write down the values below.

$x_1 = \dots, x_2 = \dots, x_3 = \dots,$

$x_4 = \dots, x_5 = \dots$

Task 8 Simplifying the function

Return to the algebra window.

Make sure that the quintic expression is highlighted and select **Factor** from the **Simplify** menu. A box giving you five options, trivial, squarefree, rational, radical and complex, should appear. Select rational and then click on the **Factor** button.

A second expression should appear with the quintic written in factorised form, a simpler form than the original form. You should be able to read off the exact values for all the roots trivially. Check these values with those estimated from the graph of the function.

Task 9 Asymptotes of a function.

Again tidy everything up, removing any expressions and graphs you have generated. Enter the function.

$$(x^3 + x^2 - 4x - 3)/(x^2 - 4)$$

If you cannot remember how to do this, look back at tasks 2 and 7. This curve has two vertical asymptotes (are you able to see what they are from the form of the function?)

Plot out the function as before.

Experiment with the scale to get a good view of the curve. You should see that the curve is divided into three separate pieces. It crosses the x -axis at three distinct points and the y -axis at one point. Use the trace cursor to find the positions of the two vertical asymptotes.

Position of the first asymptotes is $x =$

Position of the second asymptote is $x =$

Check using the trace cursor that the curve crosses the y -axis at $y = 0.75$. Again using the trace cursor find the three positions where the curve crosses the x -axis. Hence list the three roots of $(x^3 + x^2 - 4x - 3)/(x^2 - 4) = 0$

First root is $x =$

Second root is $x =$

Third root is $x =$

Return to the algebra window and use solve to find the roots of the function algebraically. Derive should give you three complicated looking expressions. Numerical approximations to these expressions can be obtained by highlighting and selecting **Approximate** from the **Simplify** menu. List the results below.

Derive calculates the roots to be $x =$, $x =$ and
 $x =$

Compare these answers with the ones you obtained graphically.

You should see that the curve has a local maximum and a local minimum. Find the coordinates of these points to one decimal place.

The position of the local maximum is (....., ..).

The position of the local minimum is (....., ..).

Looking at the curve you should notice that it approaches a straight line as $x \rightarrow \pm\infty$, that is, it has another asymptote in addition to the two vertical ones discussed above. Looking at the function for large positive and negative x , satisfy yourself that it approaches the line $y = x + 1$. To get a better idea of this, return to the algebra window and enter the new expression

$x + 1$

Then return to the plot screen and plot. You should now see both your original curve and the line $y = x + 1$. You should see that the curve really does approach $y = x + 1$ as $x \rightarrow \pm\infty$.

Task 10 (omit if short of time)

Investigate the behaviour of the function $(x^3 - 2x^2 + 2)/(x - 2)$ using Derive. Check out any local maxima and minima, real roots and asymptotes. How does the function behave as $x \rightarrow \pm\infty$.

Task 11 Differentiating a function.

- Enter the expression $x^3 + 5x^2 + 4x - 9$. If you cannot remember how to do this, then look back at Task 2.
- Select **Differentiate** from the **Calculus** menu. A window should appear.
(make sure that the variable of differentiation is set to x and the number of times the function is to be differentiated, ie the order, is set equal to 1)
- Click on **Simplify** or **OK** within this window. If you click **OK** Derive displays $\frac{d}{dx}$ of the function and you then have to select **Basic** from the **Simplify** menu.

Check that your final answer is the derivative of expression 1.

Task 12 Solving for stationary points.

Stationary points occur when $f'(x) = 0$. Therefore, we can find the stationary points of $x^3 + 5x^2 + 4x - 9$ by highlighting the expression for its derivative and selecting **Expressions** from the **Solve** menu. Select **Algebraically** and **Complex** and click the OK button. Use **Simplify Basic** to produce the two exact roots. Approximations to these solutions can be found by highlighting the expression and selecting **Approximate** from the **Simplify** menu and then clicking on **Approximate** within the window that appears.

Write down the values you obtain below.

The values of x at the stationary points are $x = \dots\dots\dots$ and $x = \dots\dots\dots$
3 decimal places.

Task 13 Classifying the stationary points.

In order to classify whether the stationary points found in Task 3 are maxima or minima we need to find the second derivative of the original function and evaluate it at each stationary point in turn. If the second derivative is positive then a minimum occurs, whereas if the second derivative is negative a maximum occurs.

Highlight the original expression for the function. Select **Differentiate** from the **Calculus** menu. Change the **Order** to order 2. Click on **Simplify**. An expression for the second derivative should appear.

To evaluate the second derivative at each stationary point in turn:

- Highlight the expression for the second derivative.
- Select **Variable Substitution** from the **Simplify** menu. A window should appear.

The variable we are replacing is indicated as x , for functions of more than one variable we can use this option to select other variables

- Enter the value of x for the first turning point *see note below*.
- Click on **Simplify** and the value of the second derivative at the given x should appear.

- This may appear as a nasty fraction; to approximate this expression select **Simplify**, **Approximate** and then **Approximate** again to give the value.
- a positive results indicates a local minima and a negative value a local maxima. Zero tells us nothing.

Note: Use the mouse buttons to highlight the required value of x , left click into the input box and then click the right mouse button and select the option to transfer the value - quicker than using the keyboard.

Repeat this exercise for the second turning point and hence find

$$x_{MAX} = \dots\dots\dots\text{and} \quad x_{MIN} = \dots\dots\dots$$

Use the **Variable Substitution** facility to evaluate $f(x)$ at each turning point.

Hence,

$$y_{MAX} = \dots\dots\dots\text{and} \quad y_{MIN} = \dots\dots\dots$$

Task 14 Plot the curve

Plot a graph of the above function and check that your analysis is indeed correct. ie you have correctly located and classified the turning points.

Task 15 Integrating a function

- Highlight the original function $f(x) = x^3 + 5x^2 + 4x - 9$.
- Select **Integrate** from the **Calculus** menu.
- You are given the option of calculating either a definite or an indefinite integral. First select the **Indefinite** option and click on **Simplify** or **OK**.

*As with the derivative, Derive either calculates the answer straight away, if you use **Simplify**, or displays \int of the function if you click **OK**. In the latter case you then have to evaluate using **Basic** from the **Simplify** menu.*

Check that the result given by Derive agrees with what you would expect. Notice that the options set the constant of integration equal to zero, strictly speaking you should always include one when you find an indefinite integral.

Now try evaluating a definite integral, for example $\int_{-5}^6 x^3 + 5x^2 + 4x - 9 dx$. Again highlight the function $f(x) = x^3 + 5x^2 + 4x - 9$. Select **Integrate** from the **Calculus** menu. Select **Definite** and insert the value 6 for the upper limit and -5 for the lower limit. Click on **Simplify**.

An answer will appear, although you may need to use **Approximate** from the **Simplify** menu to obtain a decimal result. Write your answer below.

$$\int_{-5}^6 x^3 + 5x^2 + 4x - 9 dx = \dots\dots\dots$$

Check that this is the the expected result.

Task 16 A few additional exercises.

Find the turning points of the following functions using differentiation. Check your results are reasonable by plotting the curves.

1. $f(x) = x + \frac{1}{x}$

2. $f(x) = \frac{2}{x} + \ln x$

Evaluate the following integrals.

1. $\int_4^6 \frac{13x^2 - 8x + 5}{5x^2 - 16x + 3} dx = \dots\dots\dots$

2. $\int_6^3 \frac{(x-3)^3}{(x-2)^4} dx = \dots\dots\dots$

You might like to consider how you would evaluate these integrals analytically - more of this in the Maths course.

1.1 Assessment Exercises Worksheet 1

Surname:.....

Forename:.....

Group No¹.

--

For the Actuarial Science students this sheet is to be handed in at the associated lab session. For Maths students it should be handed in at the second of the two associated lab sessions. Late submission is only allowed with the permission of the lab tutor.

1. For task 7 write down the values of x where the curve cuts the x axis:

$$x_1 = \dots\dots\dots, \quad x_2 = \dots\dots\dots,$$

$$x_3 = \dots\dots\dots, \quad x_4 = \dots\dots\dots,$$

$$x_5 = \dots\dots\dots$$

2. For task 12 write down the expression for the derivative of $f(x)$.

$$f'(x) = \dots\dots\dots$$

and correct to three decimal places the value of x at the stationary points;

$$x = \dots\dots\dots \text{and } x = \dots\dots\dots$$

3. From task 15 complete, correct to three decimal places

$$\int_{-5}^6 x^3 + 5x^2 + 4x - 9 \, dx = \dots\dots\dots$$

¹Actuarial Science Students only