## Computational Mathematics/Information Technology

## Worksheet 4

When you are asked for a percentage, give it to 2 decimal places. You may have to alter the format of the cell using the appropriate action from the toolbar.
Brief notes on the financial functions can be found on pages 3 and 4 of this handout.

## Task 1

For older versions of Excel you cannot automatically access some of the financial function. In this case do as follows:

- Select Tools from the top menu
- Select Add-Ins
- Tick Analysis ToolPak

This allows you to use the CUM functions. You should not need to do this in the computer labs.

## Task 2

I open a new savings account with $£ 100$ and then continue to invest $£ 25$ per month thereafter.
(a) If the monthly rate is $0.30 \%$ how much do I have in the account after five years?
(b) If the monthly rate had been $0.50 \%$ how long, to the nearest month, would it take for the account to accrue at least $£ 1000$ ?
(c) If I wish to accrue $£ 1500$ in 4 years what interest rate should I be looking for?
(d) If the rate and term of my investment are as in (a) by how much should I increase my monthly payments to accrue $£ 2500$ ?
(e) Again if the rate and term are as in (a) and I still wish to save $£ 25$ per month, by how much should I increase my initial investment of $£ 100$ to accrue $£ 2000$ by the end of the five years?

## Task 3

Sunil Spender a City University student decides to take out a loan to finance his latest purchase of a $4^{t h}$ generation mobile phone with built in widget connector. The loan is for $£ 500$ and he decides that he can afford to pay back no more than $£ 20$ per month for as long as it takes. If the monthly rate is $1.75 \%$ how long does it take for him to make the repayment?

## Task 4

If I borrow $£ 1200$ and pay back $£ 40$ per month for 3 years and my brother borrows $£ 2400$ and pays back $£ 80$ per month for 3 years, in terms of interest rates who has the better deal? Looking at Excel's formula for its financial functions is this the result you expected?
If in the above if my brother's loan had been over fours years, by how much worse, in terms of the difference in monthly interest rates, would his loan had been.

## Task 5

On the first of February I take out a loan for $£ 50000$ at $0.40 \%$ per month making payments of $£ 1000$ per month at the end of each month until the loan is repaid.

- How long will it take to pay off the loan. The function NPER will not return an integer value thus we can do one of two things at the end. We can either make a smaller final payment at the end of the $n^{t h}$ period or make a larger final payment at the end of the previous period. Consider each end case scenario giving in both cases the amount of the final payment.
- In this problem since equal payments do not render the loan to zero it is not possible to calculate the cumulative interest over a given period of the loan using the CUM functions, however we can us a series of IPMT functions with different $i$ values and then sum the result. If the end scenario is that of making a larger earlier payment calculate the total amount of interest paid during the life of the loan as follows:
- Set up a column of numbers from 1 to $n$, where $n$ is the total number of payments made, starting at A3; label the head of this column in cell A1 as $i$ (you may of course do this anywhere on the worksheet)
- Use the extra amount paid in the final payment as the FV value (make sure its entered as a negative).
- In B3 enter the function $=\operatorname{IPMT}(0.4 \%$, A3, your $n, 50000$, your FV, 0$)$. Copy down to include all $i$ values. Sum this column to calculate the total interest.
- To check that this calculation is indeed using the correct payments of $£ 1000$, in C3 enter the corresponding function PPMT for the principal paid and copy down to include all $i$. In each case the sum of the interest paid and the principal paid off should sum to $£ 1000$. Also the sum of the column of principals should give $-\{£ 50000+\mathrm{FV}\}$.
- If the tax year is assumed to run from April to March inclusive calculate the interest paid and the amount of loan paid off in the first complete tax year of the loan.
(You may wish to enter a column of months alongside the interest payments, column E say, to identify the tax years; do this using the auto-fill method starting with Februaryin E3 and March in E4, highlight both cells and use the + handle to copy down.)
- Calculate the interest paid in the final tax year of the loan still assuming the larger payment final scenario.


## Task 6

My Barclaycard, which I haven't used for a couple of months, is almost maxed out and I decide to cut it up and pay off my debts. I have $£ 2500$ owing and I decide to pay it off as quickly as I can, however I am not able to pay more than $£ 100$ per month. The monthly interest charge on the card is $1.75 \%$. Calculate how many equal payments I should make on each due date to pay off the loan, and exactly how much these payments should be in order to meet my budget requirements. Calculate the amount of interest I have paid off at the end of the first month and how much interest I pay in the final payment.

A friend asks, how much did the loan cost me; calculate the total amount of interest charged over the complete life of the loan using the CUMIPMT function.
In this problem it is more accurate to set the "type" parameter $=1$

## Task 7

When items are bought and they depreciate because of wear and tear and general inflation then we can use the above formulae to calculate the reduced final value by using a negative interest rate.
If I buy a car for $£ 5000$ (equivalent to investing $£ 5000$ ) and it depreciates at the rate of $0.50 \%$ per week how much will the car be worth in 2 years time (you may assume there are 104 weeks in two years)

## Supplementary Notes for Worksheet 4

The notation used in describing the various functions is as follows:

## Notation

$$
\begin{aligned}
n & =\text { total number of periods; } \quad \text { worksheet function NPER } \\
r & =\text { interest rate per period; worksheet function RATE } \\
\text { PMT } & =\text { value of each payment made during the transaction } \\
\mathrm{PV} & =\text { present value } \\
\mathrm{FV} & =\text { future value } \\
\text { type } & =0 \text { if the payments are at the end of each period. } \\
\text { guess } & \text { starting value for iterative RATE function, set }=0 \\
i & =\text { index of a period within the transaction } \\
\text { start } & \text { period number at the start of a time period within the transaction } \\
\text { end } & \text { period number at the finish of a time period within the transaction }
\end{aligned}
$$

## Functions 1

The following functions are based on the single formula derived from the problem of making an initial deposit and then making further deposits at the end of each of a given number of periods. In terms of Excel's sign convention this is given by:

$$
\begin{equation*}
\mathrm{FV}=-\mathrm{PV}(1+r)^{n}-\mathrm{PMT}\left\{\frac{(1+r)^{n}-1}{r}\right\} \tag{1}
\end{equation*}
$$

Excel has five separate functions based on this formula corresponding to making each of the variables in turn the subject of the formula. It is possible to do this explicitly for all but the rate $r$. For $r$ Excel uses an iterative process with initial "guess" set by the user or if omitted set equal to zero by Excel; this default value of zero is usually successful.

```
=FV(r, n, PMT, PV, type)
=PV(r, n, PMT, FV, type)
=PMT(r, n, PV, FV, type)
=NPER(r, PMT, PV, FV, type)
```

$=$ RATE (n, PMT, PV, FV, type, guess) uses iterative process with first value=guess

## Functions 2

Each payment, PMT, in a loan can be thought of as consisting of two parts; one part paying the current amount of interest charged and the remainder paying off some of the initial loan or principal. Excel has two functions that calculate these values for the $i^{t h}$ payment:
$=\operatorname{IPMT}(\mathrm{r}, \mathrm{i}, \mathrm{n}, \mathrm{PV}, \mathrm{FV}$, type $) \quad$ interest during the $i^{\text {th }}$ period
$=\operatorname{PPMT}(\mathrm{r}, \mathrm{i}, \mathrm{n}, \mathrm{PV}, \mathrm{FV}$, type $) \quad \begin{aligned} & \text { principal paid off during the } i^{\text {th }} \text { period }\end{aligned}$
These are based on the formula

$$
\underline{\mathrm{IPMT}}=-\left\{(1+r)^{i-1}\{\mathrm{PV} r+\mathrm{PMT}\}-\mathrm{PMT}\right\} \quad \text { and } \quad \underline{\mathrm{PPMT}}=\mathrm{PMT}-\mathrm{IPMT}
$$

The functions first calculate the PMT value before using these formula to calculate the actual values of interest paid and principal paid off in a given payment. Note that there is an option
of setting FV to some value other than zero, thus these functions can be used in a loan problem where the original loan is not completely paid off.

## Functions 3

Additionally Excel has two other functions that sum IPMT and PPMT for $i=$ start to $i=$ end. However these functions do assume that $\mathrm{FV}=0$, i.e. the loan is always paid off by the end of the $n$ periods.
$=$ CUMIPMT(r, n, PV, start, end, type) interest from "start" to "end" period inclusive $=$ CUMPRINC(r, n, PV, start, end, type) principal paid off from "start" to "end" inclusive

These two functions carry out the following calculations:

$$
\begin{aligned}
\text { CUMIPMT(r, n, PV, start, end, type }) & =\sum_{i=s t a r t}^{i=e n d} \operatorname{IPMT}(\mathrm{r}, \mathrm{i}, \mathrm{n}, \mathrm{PV}, 0, \text { type }) \\
\text { CUMPRINC(r, n, PV, start, end, type }) & =\sum_{i=\text { start }}^{i=\text { end }} \operatorname{PPMT}(\mathrm{r}, \mathrm{i}, \mathrm{n}, \mathrm{PV}, 0, \text { type })
\end{aligned}
$$

These are useful for calculating the total amount of interest paid during a loan or during part of a loan, say over a given tax year.

## Assessment Exercises Worksheet 4

Surname: $\qquad$ Forename $\qquad$
$\square$

For the Actuarial Science students this sheet is to be handed in at the associated lab session. For Maths students it should be handed in at the second of the two associated lab sessions. Late submission is only allowed with the permission of the lab tutor.

1. For Task2(a) an acceptable way of writing out the solution is:

The final value is given by: $=\mathbf{F V}(\mathbf{0 . 3 \%}, \mathbf{6 0},-\mathbf{2 5},-\mathbf{1 0 0}, \mathbf{0})=£ \mathbf{1 7 6 0 . 4 8}$
Complete the following for (b) - (e) showing the financial function used along with its parameters.
(b) = ...................................... = $\qquad$
(c) = ........................................ = $\qquad$
(d) $=\ldots . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . ~=~ . . . . . . . . . . . . . . . . . . . . . . . ~ i n c r e a s e ~=~$
(e) = ......................................... =

```
increase =
```

2. For Task 4, and again writing out the financial functions used, complete the following, giving interest rates to 2 decimal places:

The interest on my loan = $\qquad$
$\qquad$

The interest on my brother's loan = $\qquad$ $=$ $\qquad$

The interest on my brother's loan over the longer period
= .......................................... = ........................
3. For Task 6:

The number of payments $=$ $\qquad$
$\qquad$
(this will not be a whole number of payments)

Hence the whole number of payments $=$ $\qquad$
Using this value for the whole number of payments calculate the actual amount of each payment:
= ....................................... = ......................
In this type of problem the payments are at the start of each period thus the "type" parameter should be set equal to 1

[^0]
[^0]:    ${ }^{1}$ Actuarial Science Students only

