Stochastic Models 2004/05
Coursework 2

Coursework made available in the week beginning 21 March.
Suggested submission date: 7 April; deadline 14 April

1. A domestic property insurance company sells 30 policies per day, receiving a premium of £110 for each sale. The premium covers one year's worth of insurance. Each policy gives rise to an average of 1 claim per 20 years; the sizes of any claims received may be assumed to be independent gamma random variables with parameters $\alpha = 0.8$, $\beta = 0.0004$. The company's initial surplus is u.

(i) Calculate the expected amount paid out in claims per day.

(ii) Construct a simulation of the surplus process covering at least 14 days of the company's operation. You should do at least 20 simulations (using different seeds), in each case recording the minimum value attained by the surplus process over the first 14 days.
(I expect you to submit, for one simulation, a printout showing the times of the claims and the company's surplus just before and after each claim arrives, and a printout of a data table showing the minimum value for each of the 20 runs.)

(iii) In what proportion of the simulations would the company have become bankrupt if (a) $u = £0$, (b) $u = £10,000$, (c) $u = £20,000$, (d) $u = £50,000$?

(iv) Perform one simulation, for the case $u = £25,000$, to cover a period until the time of the 50th claim and submit a printout of a chart showing the path taken by the process.

2. A person who is not married (state N) at age t has a probability $0.1dt$ of marrying in the time interval $(t, t+dt)$; a person who is married (state M) at time t has probability $0.075dt$ of becoming divorced or widowed in the interval $(t, t+dt)$. All people, whether married or not, have probability $0.01dt$ of dying (entering state D) in the time interval $(t, t+dt)$. (Note: time is measured in years.)

(i) Regarding this situation as a time-homogeneous Markov jump process on state space $\{N, M, D\}$, write down the generator matrix, Q.

(ii) Write down the Kolmogorov Forward equations for $p_{NN}(t)$, $p_{NM}(t)$ and $p_{ND}(t)$.

(iii) Explain why $p_{NN}(t) + p_{NM}(t) + p_{ND}(t) = 1$.

(iv) Show that $p_{ND}(t) = 1 - e^{-0.01t}$ and hence calculate $p_{NN}(t)$ and $p_{NM}(t)$.

(v) Verify that the eigenvalues of Q are $\lambda = 0$, $\lambda = -0.01$ and $\lambda = -0.185$.

(vi) Find an expression for $P(t)$ in the form $V \exp(t\Lambda)V^{-1}$ and check that this agrees with your solution to the KFEs.

3. (Institute & Faculty of Actuaries, Subject 103, 15 September 2003)
A continuous-time process with three states is observed from time 0 up until the time of the 20th transition. The results may be summarised as follows:

State, i	No. of visits to state i	Minutes spent in state i	Number of transitions from state i to: State 1	State 2	State 3
1	8	48	0	3	5
2	4	160	1	0	3
3	8	240	7	1	0

(i) Suppose that a Markov jump process model is to be fitted to the data set above. List all the parameters of the model.

(ii) Estimate the paremeters of the model and write down the estimated generator matrix.

(iii) Suggest **one** test which could be applied as part of the model verification process. State the null hypothesis, H_0, identify the test statitsic and name its distribution u8nder H_0.

(iv) The 20th transition of the observed process takes it into state 1. Use the estimated parameter values to give point estimates of the times until the 21st and 22nd transitions.

4. A pension provider models the future lifetime of its pensioners using the following model: the probability that an individual aged t dies before reaching age $t + dt$ is $\mu(t)\,dt + o(dt)$, where $\mu(t)$, the *force of mortality*, is given by $\mu(t) = a + bt$.

(i) If this is regarded as a two-state time-inhomogeneous Markov model on state space $\{\text{Alive, Dead}\}$, write down the generator matrix $Q(t)$.

(ii) Show that

$$p_{AA}(s,t) = \exp\left(-\int_s^t (a + bu)\,du\right)$$

and complete the other entries of the transition matrix $P(s,t)$.

(iii) According to the life tables for male pensioners (PMA92C20), $\mu(65) = 0.0055$ and $\mu(85) = 0.0921$. Calculate suitable values for a and b.

(iv) Use the inverse distribution function method to simulate the time of death of a pensioner currently aged 70, using the values for a and b derived above and using a uniform pseudo-random number $U = 0.6541$.