Trade and Linked Exchange; Price Discrimination Through Transaction Bundling

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Abstract

In this paper we try to explain how price discrimination can cause bilateral trade patterns of the type seen under countertrade agreements. We interpret countertrade as a form of transaction bundling, which can discriminate between potential trading partners, and we combine characteristics from two explanations as to the existence of countertrade: Price discrimination through transaction bundling, and adverse selection arising from the uncertainty in the quality of the goods produced by trading partners in a less developed country (LDC) leading to a partner preference from the side of the Western (DC) firm. Our paper shows that the trade volume prospects of a firm in a LDC can be considerably enhanced if a countertrade transaction is bundled, and that such gains in trade become greater (relative to the case of no bundling), the greater the degree of quality uncertainty in the good it sells. It is also shown that it is profit maximising for a firm in a DC to offer mixed bundling for the exchange transaction, and that the profits derived from such bundling are a decreasing function of both the degree of uncertainty in the good sold by the firm in the LDC, and the marginal cost of the good sold by firm in the DC.

KEYWORDS: Transaction bundling, partner preference, linked exchange, countertrade, price discrimination

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1 Introduction

Countertrade agreements comprise over 20% of the international trade. Numerous examples of countertrade can be found in Hammond (1990). In a series of articles during the 90’s countertrade in its various formats (barter, buybacks, and counterpurchase - the latter being the most prevalent form of countertrade) has been examined and a number of alternative explanations offered as to its existence.

A traditional explanation for the existence of countertrade was that of foreign exchange shortages experienced by a firm in a less developed country (LDC). However, as Caves and Marin (1992) have shown empirically the traditional foreign exchange argument does not fully explain the increasing prevalence of countertrade agreements.

The second explanation refers to the moral hazard problems arising from the existence of informational asymmetries. The models by Chan and Hoy (1991) and Choi and Maldoom (1992) analyse buyback contracts as a commitment device which can assist in resolving the problem of technology transfer to developing countries by a Western firm. A buyback contract requires the Western firm to get a specified quantity of future production of the manufactured item to the production of which it has contributed by being the foreign supplier of production technology and equipment. Hence its quality decision has an impact on the quality of the good it is paid by, thus reducing its incentive to cheat by providing low quality production technology. Marin and Schnitzer (1995, 1998) generalised this result to all forms of countertrade, arguing that the tying of two transactions solves a two fold problem: it serves as a collateral and thus reduces the risk of default by the LDC firm, and it also deters cheating on quality by the firm in the developed country (DC) as the value of the collateral depends on its own quality decision (either through the existence of a technological relationship as in the case of buybacks, or by contract design). In a subsequent article on barter, Marin and Schnitzer (2002) re-emphasize the importance of paying an import with export goods as it reduces the anonymity of the medium of exchange and hence improves the LDC trading partner’s creditworthiness. In their model it is the Western firm which is unable to determine the quality of the good it is paid with if she has no experience with it, and may thus end up being paid with low quality goods. In other words, since there is uncertainty surrounding the quality of the good provided by the LDC partner, the informational asymmetry is a problem faced by the Western firm. As they argue, this is less of a problem if the good is not differentiated (more easy to obtain information about its physical and market characteristics), or if the Western firm actively contributes in the good’s production by making some costly investment on it (this latter approach re-introduces buyback characteristics into the transaction but this time as a way to resolve an informational asymmetry suffered by the DC partner, not the LDC one).

The third explanation for the existence of countertrade is based on price discrimination. Caves (1974) provides an informal explanation for the existence of countertrade agreements based on price discrimination, while Caves
and Marin (1992) have empirically verified this informal explanation; nevertheless they have not explained how it is important in understanding countertrade, nor presented a theoretical model. The main thrust of the argument is that countertrade can be used as a vehicle for price discrimination; the DC exporter can sell its good to trading partners in LDCs that will usually have low *a priori* reservation prices. As the good imported from the LDC partner in return is normally not exported and of unknown quality, the effective price paid for the export good is obscure to third parties.

Our paper tries to explain how price discrimination can cause bilateral trade patterns of the type seen under countertrade agreements. Our theoretical model is related to the bundling model of Adams and Yellen (1976), McAfee, McMillan and Whinston (1989, henceforth MMW) and the tying model of Whinston (1990). We interpret countertrade as a form of transaction bundling which can discriminate between potential trading partners and we combine characteristics from both the second and the third explanation; in our theoretical framework there is both price discrimination and uncertainty about the quality of the goods produced by LDC partners. This second feature introduces what we refer to in our model as partner preference from the side of the Western firm. A firm which is subject to a lemons type problem may be able to signal its type if it is offered the opportunity to engage in a linked auxiliary transaction which will effectively remove the anonymity of the transaction. This requires that the surplus generated by the auxiliary transaction is appropriately related to the type of the firm, thus allowing Spence - signalling to occur. Thus by offering the opportunity of bundling the transactions, the DC firm can effectively overcome its inability to identify desirable trading partners. Indeed as we show in this paper, the bundling option is profit maximising for the DC firm and the volume of trade in both goods traded increases. While our paper does confirm the assertion by Marin and Schnitzer (2002) that the extent to which barter can serve as a way to help the LDC partner overcome its creditworthiness constraint is limited by the degree of uncertainty in the quality of the good it produces, we show that barter is only a special case in exchange bundling and derive the conditions under which this special case is an optimal (i.e. profit maximising) choice for the DC partner. We then however proceed to show that the increase in the volume of trade in the good sold by the LDC partner is greater by bundling the exchange (as compared to no bundling), the greater the degree of partner preference in this good is. The main difference that allows us to do this is because we view exchange bundling as a form of price discrimination, while the Martin and Schnitzer article does not and only considers the case of barter.

### 2 The Model

We consider a model which is similar to that of MMW; there are two goods for potential trade, which are traded between a LDC market of firms, called M, and a firm P in a DC, who is either a monopolist or a monopsonist or both. There is
a continuum of firms in M, which vary in quality terms and each may choose to transact in neither of the two goods, just one good, or both goods. P is unable to identify the type of a particular firm in M before transacting with it.

The model allows transactions to occur in any direction between M and P. Each firm in M has a unit demand (supply) for each good it is buying (selling). \((\theta_1, \theta_2)\) are the set of valuations placed on the two goods by the firms in M, with \(\theta_1\) and \(\theta_2\) having independent uniform distributions over \([0, 1]\). Each firm’s surplus is equal to

\[ S_i(\theta_1, \theta_2) = \theta_i(-\theta_i) \]

If good \(i\) is bought (sold) by a firm in M, then \(\theta_i\) is the value (cost of production) to that firm. Firm P’s surplus gross of payments is equal to \(S^P_i\), and for symmetry \(|S^P_i| \in [0, 1]\). \(S^P_i\) is equal to the cost of production if P sells good \(i\), i.e. \(S^P_i = -c_i\). On the other hand, if P buys the good, \(S^P_i = v(\theta_i)\), where \(v(\theta_i)\) is the valuation P assigns to the good. In this second case, P splits its valuation of the good into the sum of two components: the trading partner’s choice of quality expressed as a one to one function of the the trading partner’s cost of production \(\theta_i\) (henceforth referred to as the "type" of the trading partner) to which it assigns a weight of \(\beta\) (explained in more detail below), plus \(\alpha\) which is equal to some minimum quality value that this good would raise if sold in the international market\(^1\). Hence:

\[ v(\theta_i) = \alpha + \beta \theta_i \]

where \(0 \leq \alpha + \beta \leq 1\). If the quality of the good sold by the firms in M is uncertain, we say that P has a partner preference, as the latter’s surplus function depends on the quality type of the firm in M with which it transacts. The term \(\beta\) can be thought of as a measure of the intensity of P’s partner preference, i.e., the importance it assigns to (or the extent of its inability to uncover the features determining) the uncertain quality of the good. If \(\alpha = 1\), then there is no partner preference \((\beta = 0)\) and P’s net surplus function on the good it purchases will definitely be positive.

P announces three prices: \(p_1, p_2\) are the prices for the single transactions in each good, and \(p_b\) is the price offered by P for the two transactions bundled together. In other words, this is a model of mixed bundling in which P offers to either transact in each good separately or in both goods bundled together. If P sells (buys) good \(i\) then conventionally we will take \(p_i > 0\) \((p_i < 0)\).

We will assume that it is not possible for P to monitor who it transacts with. An obvious reason for this monitoring problem is that firms in the market can engage a third party to act as a middleman for either of the goods transacted and so hide their identity. The lack of monitoring opportunities means that P faces the constraint that \(p_b \leq p_1 + p_2\) for some firms to wish to engage in

\(^1\)\(\alpha\) can also be thought as the extent of P’s ability to ascertain some of the features of the good it considers buying.
the bundled transaction. In other words, we can define the discount factor \( \epsilon \) as 
\[ \epsilon = p_1 + p_2 - p_b \geq 0. \]

There will be four groups of firms in M: those who choose to transact in neither good (\( R_0(p) \)) at the set of prices \( p = (p_1, p_2, p_b) \) announced by P; those who trade in good 1 only (\( R_1(p) \)); those who trade good 2 only (\( R_2(p) \)); and those who trade in both goods (\( R_{12}(p) \)). Those firms who transact in only one good receive a surplus of \( S_1(\theta_1, \theta_2) - p_1 \), while the firms that transact in both goods get a net surplus of \( S_1(\theta_1, \theta_2) + S_2(\theta_1, \theta_2) - p_b \). Accordingly, the four groups are defined as:

\[
R_0(p) = \{ \theta \in [0, 1]^2 : S_1(\theta) < p_1, S_2(\theta) < p_2, S_1(\theta) + S_2(\theta) < p_b \};
\]

\[
R_1(p) = \{ \theta \in [0, 1]^2 : S_1(\theta) - p_1 = \max(0, S_1(\theta) - p_1, S_2(\theta) - p_2, S_1(\theta) + S_2(\theta) - p_b) \};
\]

\[
R_2(p) = \{ \theta \in [0, 1]^2 : S_2(\theta) - p_2 = \max(0, S_1(\theta) - p_1, S_2(\theta) - p_2, S_1(\theta) + S_2(\theta) - p_b) \};
\]

\[
R_{12}(p) = \{ \theta \in [0, 1]^2 : S_1(\theta) + S_2(\theta) - p_b = \max(0, S_1(\theta) - p_1, S_2(\theta) - p_2, S_1(\theta) + S_2(\theta) - p_b) \}.
\]

These four sets partition the unit square.

The net surplus for P can now be written down. It is equal to:

\[
\sum(p) = \int_{R_0(p)} (S_1^P + p_1) \, dF(\theta) + \int_{R_1(p)} (S_2^P + p_2) \, dF(\theta) + \int_{R_{12}(p)} (S_1^P + S_2^P + p_b) \, dF(\theta)
\]

The three prices must now be chosen to maximize \( \sum(p) \). If the optimal prices are such that \( p_b < p_1 + p_2 \), then P is offering more favourable terms for the linked transaction than the two separate transactions. We will refer to this as exchange transaction bundling. We will proceed to show that transaction (mixed) bundling is optimal not only locally, as in the MMW model for the monopoly case, but globally by checking both the first and the second order conditions in all three cases analysed below. However we do not follow the steps of the MMW paper here for two reasons. First, our purpose is to offer an analytical view of the case of exchange transaction bundling, rather than to look at the general framework of bundling exclusively as MMW do for the case of sellers. Second, the adoption of a general framework constraints MMW to solve the problem only in terms of local optimisation. Instead, we proceed to solve the problem by global optimisation and in order to do that we need to take a more specific case by assuming independently distributed uniform values for \( \theta_1 \) and \( \theta_2 \).
3 Bundling an exchange transaction

Firm P sells good 1 and buys good 2, whereas a firm in M buys good 1 and sells good 2. The quality of good 2 is uncertain but the quality of good 1 is certain, as the latter is produced by P (a firm in a DC). The first uncertainty parameter $\theta_1$ is the valuation given to good 1 by the firm in M. The second uncertainty parameter $\theta_2$ is the cost to the firm in M of producing good 2. As mentioned earlier P’s valuation of good 2 is equal to $\nu(\theta_2) = \alpha + \beta \theta_2$, where $0 \leq \alpha + \beta \leq 1$. P produces good 1 at cost c. Thus the gross surplus functions are given by

$$S_1(\theta_1, \theta_2) = \theta_1, S_2(\theta_1, \theta_2) = -\theta_2$$

**Theorem 1** Mixed bundling in exchange transaction is optimal for $\alpha > 0, c < 1$. If either $\alpha = 0$ or $c = 1$, then non-bundling is optimal for P. Both P’s net surplus and the bundling discount factor are decreasing functions of P’s production cost (c) and the intensity of partner preference and increasing functions of $\alpha$. The absolute values of the prices offered to the trading partner for goods 1 and 2 are increasing in $\alpha, \beta$ and c.

**Proof.** See the Appendix.

After deriving the first and second order conditions for profit maximisation we find that mixed bundling an exchange transaction is a profit maximising strategy for P, and that the profits derived from such a transaction reach a maximum value when the marginal cost of the good P sells is equal to zero and when the degree of partner preference is nil ($\alpha = 1, \beta = 0$). However, exchange bundling continues to be optimal as a strategy and therefore exercised by P even if both marginal cost and the degree of partner preference are strictly positive ($\alpha < 1, \beta > 0$). It is only when P is faced with a partner for which $\alpha = 0$ and/or the marginal cost of a good it produces is equal to one that the optimal value of $c$ was found to be equal to zero (i.e. bundling is not optimal).

In a model of exchange bundling, $\theta_2$ has a dual role: it is a participation constraint both for the firm in M as well as for firm P, as the latter’s valuation of good 2 depends on this variable; for example, in the case of no bundling M will not trade with P in good 2 unless $-\theta_2 \geq \theta_2^*$, and P will not trade with a firm in M unless $-\theta_2 \leq \alpha + \beta \theta_2$. This implies a net surplus of $\int_0^{\theta_2^*} (\alpha + \beta \theta_2 + \theta_2) d\theta_2$, which is maximized for $-\theta_2^* = \frac{\alpha}{1-c}$ (while $\theta_1^* = \frac{1+c}{2}$). Figure 1 illustrates unbundled exchange where the optimal price of $-\theta_2^*$ chosen by P (thus satisfying P’s participation constraints regarding good 2) will also satisfy the participation constraints for trading in good 2 of any firm in M which belongs in the areas $R_1^U$ and $R_{12}^U$. On the other hand, the price $\theta_1^*$ chosen by P will satisfy the valuation constraints of those firms in M which belong in areas $R_1^L$ and $R_{12}^L$ and who will purchase good 1 from firm P.
For the case of mixed bundling of the transactions of sell and buy, P’s surplus (profit) maximising values of \( p_1, p_2 \) and \( \epsilon \) are derived by solving the first and second order conditions (see the appendix). These are illustrated in Figure 2 together with the unbundled prices of \( p_1^* \) and \( p_2^* \). As \( p_1 \) and \( p_2 \) in the diagram are now the optimal individual prices offered by P in mixed bundling, they already satisfy P’s participation constraints. Figure 2 highlights the areas that also satisfy the participation constraints of the firms in M in their decision on whether to trade. We see that the areas of untied trade, while they still exist are much reduced. Firm * who previously only traded in good 2 in Figure 1 because its valuation for good 1 was rather low, will now trade in both goods in area \( R_{12}^B \). Firm *** who previously only traded in good 1 in figure 1 because its cost of producing good 2 was not covered by \( -p_2^* \), will now trade in both goods in area \( R_{12}^B \). Finally, firm ** who previously did not trade in either good in figure 1 because its valuation (cost of producing) good 1 (good 2) was smaller (greater) than the optimal unbundled prices offered by P, will now trade in both goods. It is obvious from comparing the two figures that the volume of trade in mixed exchange bundling has increased relative to the case of no bundling for both goods. P raises (reduces) the price \( p_1 \) (\( p_2 \)) of the good it sells (buys) in relation to its unbundled value \( p_1^* \) (\( p_2^* \)), thus decreasing the incentive of a firm in M to participate in untied trade within areas \( R_1 \) (\( R_2 \)); however the reduction in trade that this causes is more than offset by the increase in the size of area \( R_{12} \), i.e. the offering of a premium \( \epsilon \) for the bundled transaction is substantial enough to increase overall trade in both goods \( (R_1^B + R_{12}^B) \) and good 2 \( (R_2^B + R_{12}^B) \) relative to its unbundled size \( (R_1^U + R_{12}^U) \) and \( R_2^U + R_{12}^U \) correspondingly). It is easy to numerically check that there is always an increase in the size of trade in both goods relative to its unbundled size by substituting for the bundled and unbundled surplus profit maximising prices and the corresponding optimal value of \( \epsilon \) and calculating for these:

\[
R_1^B + R_{12}^B - R_1^U - R_{12}^U = p_1^* - p_1 - \epsilon p_2 + \frac{\epsilon^2}{2},
\]

and

\[
R_2^B + R_{12}^B - R_2^U - R_{12}^U = p_2^* - p_2 + \epsilon(1 - p_1) + \frac{\epsilon^2}{2},
\]

both of which were found to be always positive for the range of all possible values of \( \alpha, \beta \) and \( \epsilon \).

More specifically, the improvement in the trade for good 1 is an increasing function of \( \alpha, \beta \) and \( \epsilon \), with the \( \alpha \)'s effect dominating over \( \beta \)'s in the case of

\(^2\)Can be located in figure 2 as consisting of the reduction in trade for good 1 equal to the area of the rectangle with base length \( p_1 - (p_1 - \epsilon) = \epsilon \) and height \( (-p_2 + \epsilon) - (-p_2) = \epsilon \), plus the area of the rectangle with height \( -p_2 \) and base \( p_1 - (p_1 - \epsilon) = \epsilon \).

\(^3\)Can be located in figure 2 as consisting of the reduction in trade for good 2 equal to the area of the rectangle with height \( p_2^* - p_2 \) and base length of one, and the gains in trade for 2 given by the areas of the triangle with base \( p_1 - (p_1 - \epsilon) = \epsilon \) and height \( (-p_2 + \epsilon) - (-p_2) = \epsilon \), plus the area of the rectangle with height \( (p_2^* + \epsilon) - (-p_2) = \epsilon \) and base \( 1 - p_1 \).
an equal change in the values of both parameters in opposite directions. For any given value of $c$ the maximum percentage increase in the trade in good 1 is achieved when the degree of partner preference is nil (i.e. for $\alpha = 1$ and $\beta = 0$, this is equal to 20.3% for $c = 0$, increasing only fractionally even if $c$ is increased substantially, for example to 20.5% for $c = 0.6$). On the other hand, the improvement in the trade for good 2 is decreasing in $\alpha$ and $c$ and increasing in $\beta$, this time with $\beta$’s effect dominating over $\alpha$’s in the case of an equal change in the values of both in the same direction. The maximum percentage increase in the volume of trade for good 2 is achieved when $c = 0$ for all given values of $\alpha$ and $\beta$. Given that the marginal cost of producing good 1 is zero, then the maximum increase in the trade for good 2 is achieved for $\alpha (\beta)$ being as close to zero (one) as possible. It is now the minimum percentage increase in the volume of trade for good 2 which is equal to 20.3% for $\alpha = 1$, $\beta = c = 0$, while this reaches very high values for $c > 0$, low values of $\alpha$, and high values of $\beta$ (for example for $\alpha = 0.2, \beta = 0.8, c = 0$ this equals 58.1%), provided that $\alpha > 0$ (since for $\alpha = 0$, bundling is no longer optimal for P as $\epsilon$ takes the profit maximising value of zero). In other words, provided that P does find it profit maximising to engage into a bundled exchange (and it always does for $\alpha > 0$ and $c < 1$), the prospects of improving the volume of trade for the good sold by a firm in a less developed country through exchange bundling are more extensive the higher the degree of partner preference (the greater the extent of quality uncertainty) for this good is, and the lower P’s marginal cost of producing the good to be used in the linked exchange is. Hence the more pronounced the problem of informational asymmetry is for countries in LDCs (i.e. the large $\beta$ is), the more substantial is the increase in trade by having firms of type ** and *** (as indicated in Figure 2) which previously did not sell 2, now joining $R_{12}^B$.

Our model presents countertrade in the form of commodity bundling of a reciprocal exchange, and shows that the existence of such an option in addition to the offering of two separate prices by P is profit enhancing relative to the absence of such a linked exchange option, with only two exceptions as noted in Theorem 1. We combine the price discrimination approach to countertrade with that of adverse selection (arising from the informational asymmetry which results from the uncertain quality of the good sold by the firms in M) by introducing the concepts of transaction bundling and partner preference respectively. As far as P is concerned, we find support for the view that a countertrade arrangement in B2B trade can produce higher profits than an arrangement in which only untied trading in each good is offered. From the point of view of the firms in M, the more pronounced the degree of informational asymmetry is, the more firms will benefit by overcoming the problem of adverse selection arising from quality uncertainty which prevented them from selling good 2, by now becoming members of group $R_{12}^B$. It is only a smaller fraction of firms that in the absence of bundling would have traded only in good 2 ($R_{22}^B$), and with mixed bundling do not trade in either good.

Note that the gains in the trade for good 1 are only 5.1% in this case.
Finally, it is interesting to note that a value of $p_0 = 0$ indicates that special case where the countertrade transaction will take the form of a barter exchange. It can be shown that barter may occur only within a specific range of values: $\alpha \in [0.5, 1]$, $\beta \in [0, 0.5]$, $c \in [0, 0.25]$ resulting into a surplus maximising range of values for $\epsilon$ between $\left[\frac{1}{3}, \frac{2}{3}\right]$. More specifically, within this range of values $\epsilon$ is a decreasing and convex (concave) function of $\alpha$ ($\beta$ and $c$). For $\alpha = 1$, $\beta = 0$ and $c = 0.25$, barter occurs as it is optimal at $\epsilon = \frac{1}{3}$, while for $\alpha = \beta = 0.5$ and $c = 0$ barter is again optimal at $\epsilon = \frac{2}{3}$. In other words, a necessary condition for a barter contract to be a profit maximising way for firm P to conduct a bundled exchange is that the degree of partner preference does not exceed the value of a half and that P has marginal cost that does not exceed the value of 0.25. This finding supports the assertion by Marin and Schnitzer in their 2002 article that the extent to which barter can be used by developing countries to increase their volume of trade is limited by the degree of partner preference that the uncertainty in the quality of what they produce introduces. An additional limitation to this, according to our paper, is also the marginal cost of the good that P bundles into the transaction. It is only in the special cases where $\alpha$, $\beta$ and $c$ all happen to have specific values such that the profit maximising values of $p_1, p_2$ and $\epsilon$ turn out to be such that $p_0 = 0$, that barter turns out to be optimal.

However in our article barter is only a special case. We have shown that gains in the volume of trade from bundling an exchange transaction are substantial; in fact the more intense the degree of partner preference in the good sold by the firms in a LDC is, the larger this gain is. If there is no partner preference (i.e., if $\alpha = 1$ and $\beta = 0$) then the gains in the volume of trade in good 2 are exactly equal to those experienced in good 1 when $c = 0$, as the latter good is also of certain quality (always) and the overall net surplus in this good is also 1 if $c = 0$. However, as $\alpha$ decreases and $\beta$ increases the gains in the volume of trade in good 2 increase while those of 1 decrease. However, while the profit maximising discount premiums set by P for this bundled exchange decrease and so do its profits from doing so, exchange is still offered by P as it is profit maximising to do so. The only discontinuity is for the extreme case of $\alpha = 0$, in which case bundling is no longer offered (as the optimal value is $\epsilon = 0$) and hence the gains from trade in good 2 (and in good 1 as well) relative to no bundling will discontinuously drop to zero.

4 Conclusions

The purpose of this paper was to try to provide a general theoretical framework for explaining the continued prevalence of countertrade transactions. By bundling the exchange using a mixed format, the volume of trade for a firm in a developing country can be considerably increased by the introduction of bundling (by up to 58%) as compared to having the transactions conducted separately. These results are intuitively consistent as it is obvious that there is
more room for an increase in the volume of trade through bundled exchange for the good sold by a firm in a LDC (relatively to selling it separately) the larger the quality uncertainty component of the good it trades is, and the lower the marginal cost of the good that the firm in the developed country bundles into the transaction is. Since the uncertainty in the value of the good traded by the firm in a LDC acts as a participation constraint for the firm in the developing country, it implies that the profitability of bundling for the latter increases as the degree of partner preference, as well as its own marginal cost for the good it sells, both decrease.

The existence of participation constraints means that barter can only emerge as an optimal strategy for a certain range of values of marginal cost and degree of ability to assess quality; hence barter is limited in its ability to help firms in LDCs overcome the problems of informational asymmetry that the uncertainty in the quality of the good they sell introduces. As our model clearly shows this happens because barter is only but a special case of bundled exchange, since it is a case in which the bundled price is restricted to be equal to zero and this will not necessarily be a profit maximising solution for the firm that sets the prices.
5 APPENDIX

5.1 Proof of Theorem 1

The first step is to identify the sets of inequalities defining regions $R_0, R_1, R_2, R_{12}$:

- $R_0(p) = \{(\theta_1, \theta_2) \in [0,1]^2 : p_1 > \theta_1, \theta_2 > -p_2, \theta_2 - \theta_1 > -p_6\}$;
- $R_1(p) = \{(\theta_1, \theta_2) \in [0,1]^2 : \theta_1 > p_1, \theta_2 > p_1 - p_6\}$;
- $R_2(p) = \{(\theta_1, \theta_2) \in [0,1]^2 : -p_2 > \theta_2, p_6 - p_2 > \theta_1\}$;
- $R_{12}(p) = \{(\theta_1, \theta_2) \in [0,1]^2 : p_6 - p_2 < \theta_1, p_1 - p_6 > \theta_2, \theta_2 - \theta_1 < -p_6\}$.

Based on these, the net surplus to $P$ is given by:

$$
\sum = \int_0^{\theta_1-p_6} \int_0^{\theta_1-p_6} (p_1-c) d\theta_1 d\theta_2 + \int_0^{-p_2} \int_0^{p_6-p_2} (\alpha + \beta \theta_2 + p_6) d\theta_1 d\theta_2 +
\int_0^{\theta_1-p_6} \int_0^{p_1} (\alpha + \beta \theta_2 - c + p_6) d\theta_1 d\theta_2 + \int_0^{p_1} \int_0^{\theta_1-p_6} (\alpha + \beta \theta_2 - c + p_6) d\theta_2 d\theta_1
$$

Substituting $p_6 = p_1 + p_2 - \epsilon$ and evaluating the integrals in $P$’s net surplus gives:

$$
\sum = [(p_1-c)(1-p_1) - \alpha p_2 + (0.5 \beta - 1)p_2^2] + 
[\alpha (1-p_1) + (2 \beta + \epsilon)p_2 + (\beta - 3)p_1 p_2] \epsilon + 
\frac{1}{2} [\alpha + \beta - c - 2 + (3 \beta)(p_1 + p_2)] \epsilon^2 + \frac{1}{6} (\beta - 3) \epsilon^3
$$

The net surplus with no bundling, that is when $\epsilon = 0$, is given by

$$
\sum_0 = (1-p_1)(p_1-c) - \alpha p_2 + (0.5 \beta - 1)p_2^2
$$

Maximising this with respect to $p_1$ and $p_2$ ($\frac{\partial \bar{\Sigma}}{\partial p_1} = \frac{\partial \bar{\Sigma}}{\partial p_2} = 0$) gives the optimal unbundled prices $p_1^*, p_2^*$ for the two goods (while it is easy to check that the second order conditions are also satisfied):

$$
p_1^* = \frac{1}{2} (1 + c) \quad p_2^* = \frac{\alpha}{\beta - 2}
$$
By partially differentiating $\sum$ with respect to $\alpha$, $\beta$ and $c$ it is easy to show that it is an increasing function of all three parameters. In order to derive the net surplus maximising level in bundled exchange, we need to set

$$\frac{\partial \sum}{\partial p_1} = \frac{\partial \sum}{\partial p_2} = \frac{\partial \sum}{\partial \epsilon} = 0,$$

for deriving the three first order conditions:

$$\frac{\partial \sum}{\partial p_1} = c - \alpha \epsilon - 2p_1 - 3\epsilon p_2 + \beta \epsilon p_2 + \frac{3}{2} \epsilon^2 - \frac{1}{2} \beta \epsilon^2 + 1 = 0$$

$$\frac{\partial \sum}{\partial p_2} = 2\epsilon - \alpha + \alpha \epsilon - \beta \epsilon - 3\epsilon p_1 + \beta \epsilon p_1 + \frac{3}{2} \epsilon^2 - \frac{1}{2} \beta \epsilon^2 + p_2 (\beta - 2) = 0$$

$$\frac{\partial \sum}{\partial \epsilon} = \alpha - 2\epsilon - \alpha \epsilon + \alpha \epsilon + 2p_2 + \epsilon p_2 - \alpha p_1 - \beta p_2 +$$

$$+3\epsilon p_1 + 3\epsilon p_2 - \beta \epsilon p_1 - \beta \epsilon p_2 - 3p_1p_2 + \beta p_1 p_2 - \frac{3}{2} \epsilon^2 + \frac{1}{2} \beta \epsilon^2 = 0$$

Adding the second with the third F.O.C. and solving for $\epsilon$ gives:

$$\epsilon = \frac{-c p_2}{\alpha + \beta p_2 - \beta p_2} + p_1$$

while adding the first and the third F.O.C. and solving for $\epsilon$ gives:

$$\epsilon = p_2 + \frac{1}{-c + \beta + 3p_1 - \beta p_1 - 2 (\alpha - 2p_1 + \alpha p_1 - 1)}$$

While the three equations can be solved for expressing the optimal bundled prices and discount factor as functions of $\alpha$, $\beta$ and $c$, the resulting expressions are extremely long and complex and can not be interpreted; hence we do not present them here. Instead, what we do is we first check the second order conditions (SOC) to see the constraints, if any, these impose on the values of the parameters $\alpha$, $\beta$ and $c$, and then derive the optimal values of $p_1$, $p_2$ and $\epsilon$ under bundled exchange within these SOC satisfying value ranges:

$$-2 < 0$$

$$\det \begin{bmatrix} -2 & \epsilon (\beta - 3) \\ \epsilon (\beta - 3) & \beta - 2 \end{bmatrix} > 0 \iff \det \begin{bmatrix} (\beta - 3)^2 \epsilon^2 - 2\beta + 4 \end{bmatrix} > 0 \iff \epsilon < \frac{\sqrt{2} \sqrt{2 - \beta}}{3 - \beta}$$
Hence, for $\beta = 1 \Rightarrow \epsilon < 0.7071$ and for $\beta = 0 \Rightarrow \epsilon < \frac{1}{\sqrt{2}}$. Finally, the determinant of the 3x3 matrix below should be negative:

$$
\begin{array}{ccc}
-2 & \epsilon(\beta - 3) & (\beta - 3)(p_2 - \epsilon) - \alpha \\
\epsilon(\beta - 3) & \beta - 2 & (\beta - 3)(p_1 - \epsilon) - \beta + c + 2 \\
(\beta - 3)(p_2 - \epsilon) - \alpha & (\beta - 3)(p_1 - \epsilon) - \beta + c + 2 & (3 - \beta)(p_1 + p_2 - \epsilon) + \alpha + \beta - c - 2 \\
\end{array}
$$

We have used the DERIVE mathematical software to derive the optimal values of $p_1, p_2, \epsilon$ and the maximum net surplus of $P$ for different values of $\alpha$, $\beta$, and $c$. It can be seen that the optimal net surplus reaches a peak value of 0.5492 for $\alpha = 1$ and $\beta = c = 0$. Additionally we have calculated the conditions under which $p_b = 0$, i.e. the cases where barter is optimal. A barter optimal value of $\epsilon$ is an inverse and convex (concave) function of $\alpha$ ($\beta$ and $c$) and takes a maximum value $\epsilon = 0.4$ for $\alpha = \beta = 0.5$ and $c = 0$, and a minimum value of $\epsilon = \frac{1}{4}$ for $\alpha = 1$, $\beta = 0$ and $c = 0.25$. 

Unbundled exchange

Figure 1
Mixed bundled exchange

Figure 2
6 References


