Maths for Actuarial Science Coursework 1

This is an assessed coursework, and will count towards your final grade. Solutions should be handed in to the **mathematics general office** (CM520) by **2:00pm on Thursday 18th November**. Late submissions will be penalised.

1. (i) By considering the general term, find the coefficient of x^{14} in the expansion of

$$\left(x^3 + \frac{3}{x^2}\right)^{18}$$

expressing your answer as a product of primes.
(ii) Write down the expansion of $(1+x)^6$. Hence, by letting $x = z + z^2$, expand $(1 + z + z^2)^6$ in ascending powers of z as far as the term in z^4.

2. (i) Find the points of intersection of the circles

$$x^2 + y^2 - 2x - 4y - 4 = 0 \qquad \text{and} \qquad x^2 + y^2 - 6x - 2y - 8 = 0.$$

(ii) Find an equation for the ellipse with foci at $(4, 2)$ and $(6, 2)$ and major axis of length 6.

3. (i) Using the identities for $\cos A + \cos B$, and for $\cos^2 C$ in terms of $\cos 2C$, show that for any values of α, β, γ we have

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2(\alpha+\beta+\gamma) = 2(1 + \cos(\beta+\gamma)\cos(\gamma+\alpha)\cos(\alpha+\beta)).$$

(ii) Find in terms of π the general solutions to

$$\sin 5\theta + \sin\theta = \sin 3\theta.$$

4. (i) Differentiate the following functions:

$$\text{(a)} \quad (1+3x)^2 \ln(1+3x) \qquad \text{(b)} \quad \tan^2(x^3+1).$$

(ii) Given that $y = (Ax + B)e^{-2x} + 2\sin 3x + \cos 3x$ where A and B are constants, show that

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 4\frac{\mathrm{d}y}{\mathrm{d}x} + 4y = 19\cos 3x - 22\sin 3x.$$

5. Evaluate the following integrals:

$$\text{(a)} \quad \int \frac{x+2}{1-4x^2}dx \qquad \text{(b)} \quad \int x^2 e^{-2x}dx \qquad \text{(c)} \quad \int \frac{1}{2x^2-4x+10}dx.$$