Maths for Actuarial Science Coursework 1

This is an assessed coursework, and will count towards your final grade. Solutions should be handed in to the **mathematics general office** (CM326) by **2:00pm on Tuesday 7th November**. Late submissions will be penalised.

1. Solve the equation
$$\left|\frac{x-1}{x+2}\right| < \left|\frac{x+1}{x-2}\right|.$$
[10]

2. (i) Find an equation for the ellipse with foci at $(4, 2)$ and $(6, 2)$ and major axis of length 6.
(ii) Explain why, for any circle C and point P outside it, the lengths of the two tangents from C to P are always equal. **[10]**

3. Find the general solution of
$$\sin 2\theta + \sin 3\theta + \sin 5\theta = 0.$$
[10]

4. (i) Differentiate the following expressions with respect to x, simplifying where possible:
(a) $\ln(\sec x + \tan x)$ (b) $\sin^3 x \cos 3x + \cos^3 x \sin 3x$ (c) a^x where $a > 0$.
(ii) Find the first and second derivatives (with respect to x) of the function
$$x = 4t^2 - 5t + 6 \qquad y = t^3 - t^2 + t.$$
[10]

5. Evaluate the following integrals:
$$\text{(a)} \quad \int \frac{x^3}{(x^2-9)(x+1)}\,dx \qquad \text{(b)} \quad \int x^2 e^{-2x} dx.$$
[10]