

## Maths for Actuarial Science Coursework 1

This is an assessed coursework, and will count towards your final grade. Solutions should be handed in to the **mathematics general office** (C123) by **3:00pm on Thursday 8th November**. Late submissions will be penalised.

1. (i) By considering the general term, find the coefficient of  $x^{14}$  in the expansion of

$$\left(x^3 + \frac{3}{x^2}\right)^{18}$$

expressing your answer as a product of primes.

- (ii) Write down the expansion of  $(1+x)^6$ . Hence, by letting  $x = z + z^2$ , expand  $(1+z+z^2)^6$  in ascending powers of  $z$  as far as the term in  $z^4$ . [7]

2. (i) Find the points of intersection of the circles

$$x^2 + y^2 - 2x - 4y - 4 = 0 \quad \text{and} \quad x^2 + y^2 - 6x - 2y - 8 = 0.$$

- (ii) Find an equation for the ellipse with foci at  $(3,5)$  and  $(3,7)$  and major axis of length 6. [10]

3. (i) Using the identities for  $\cos A + \cos B$ , and for  $\cos^2 C$  in terms of  $\cos 2C$ , show that for any values of  $\alpha, \beta, \gamma$  we have

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma + \cos^2(\alpha + \beta + \gamma) = 2(1 + \cos(\beta + \gamma) \cos(\gamma + \alpha) \cos(\alpha + \beta)).$$

- (ii) Find in terms of  $\pi$  the general solutions to

$$\sin 5\theta + \sin \theta = \sin 3\theta.$$

[10]

4. (i) Differentiate the following functions:

$$(a) \quad (1 + 3x)^2 \ln(1 + 3x) \quad (b) \quad \tan^2(x^3 + 1).$$

- (ii) Find the second derivative (with respect to  $x$ ) of the function

$$x = t^2 + t + 1 \quad y = \ln(t).$$

[8]

5. Evaluate the following integrals:

$$(a) \quad \int \frac{x+2}{1-4x^2} dx \quad (b) \quad \int_0^1 \frac{2x}{\sqrt{2x+1}} dx.$$

[8]

6. Express  $\sin(2 \tan^{-1} x)$  in terms of  $x$  only. [7]