

Maths for Actuarial Science Coursework 1

This is an assessed coursework, and will count towards your final grade. Solutions should be handed in to the **mathematics general office** (C123) by **3:00pm on Thursday 12th November**. Late submissions will be penalised.

1. Solve the equation

$$\left| \frac{x-1}{x+2} \right| < \left| \frac{x+1}{x-2} \right|.$$

[10]

2. (i) Find an equation for the ellipse with foci at $(3, 5)$ and $(3, 7)$ and major axis of length 6.

(ii) Explain why for any circle C and point P outside it, the lengths of the two tangents from C to P are always equal.

[10]

3. (i) Using the identities for $\cos A + \cos B$, and for $\cos^2 C$ in terms of $\cos 2C$, show that for any values of α, β, γ we have

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma + \cos^2(\alpha + \beta + \gamma) = 2(1 + \cos(\beta + \gamma) \cos(\gamma + \alpha) \cos(\alpha + \beta)).$$

(ii) Find in terms of π the general solutions to

$$\sin 5\theta + \sin \theta = \sin 3\theta.$$

[10]

4. (i) Differentiate the following functions:

$$(a) \quad (1 + 3x)^2 \ln(1 + 3x) \qquad (b) \quad \tan^2(x^3 + 1).$$

(ii) Find the second derivative (with respect to x) of the function

$$x = t^2 + t + 1 \qquad y = \ln(t).$$

[10]

5. Evaluate the following integrals:

$$(a) \quad \int \frac{x+2}{1-4x^2} dx \qquad (b) \quad \int_0^1 \frac{2x}{\sqrt{2x+1}} dx \qquad .$$

[10]