
Mathematics for Actuarial Science (AS1051)

2010

*Full marks can be obtained by answering all six questions.
All necessary working must be shown.*

TIME ALLOWED: 90 minutes

- 1) [6 marks] Find the coefficient of x in expansion of the expression

$$\left(x^5 + \frac{2}{x^3}\right)^{13}$$

by considering the general term. Express your answer as a product of primes.

- 2) [9 marks] An ellipse is parameterized by the equation

$$2x^2 + 3y^2 - 4x + 5y + 4 = 0 .$$

In the x, y -plane find the centre of the ellipse, the length of the major and minor axis, the location of the foci, its eccentricity and the equation of the directrix.

- 3) [9 marks] For the two parametric functions

$$x(t) = 4t^2 + 3 \quad \text{and} \quad y(t) = \ln(2t^2 + 1)$$

find the function $q(t)$ such that

$$\frac{d^2y}{dx^2} = q(t) \frac{d^2y}{dt^2} \frac{dt^2}{dx^2} .$$

- 4) [6 marks] Prove by induction that the n^{th} derivative of the function

$$y = \sin(ax)$$

is given by

$$\frac{d^n y}{dx^n} = a^n \sin\left(ax + \frac{n\pi}{2}\right) .$$

Turn over ...

5) [11 marks] Compute the integrals

$$i) \int \cos 5\theta \cos 4\theta d\theta \quad \text{and} \quad ii) 9 \int x^2 e^{3x} dx.$$

6) [9 marks] Compute the first two non-zero terms of the Taylor expansion for the function

$$f(x) = \sin x \ln(1 + x)$$

about $x = 0$.