Mathematics for Actuarial Science (AS1051)

2011

Full marks can be obtained by answering all six questions.
All necessary working must be shown.

TIME ALLOWED: 90 minutes

1) [6 marks] Solve the following set of equations

$$7x + y - 2z = 4,$$
$$-20w + 2x - y + z = 2,$$
$$w - x + 3y + z = -5,$$
$$-7w - 2x + y + 2z = 1,$$

for w, x, y and z.

2) [9 marks] Solve the equation

$$7\cosh\theta - 2\sinh^2\theta = 8$$

for θ. Express your answer in terms of logarithmic functions.

3) [9 marks] A parabola is parameterised in the form

$$x^2 + y^2 = (8 + x)^2 \ .$$

Find a transformation $(x, y) \to (X, Y)$ which converts the parabola into its normal form $Y^2 = 4aX$ and determine the constant a. In the (X, Y)-coordinate system find the points of intersection between the parabola and the line $\mathcal{L}$ passing through the points $(0, 1)$ and $(2, 7)$. At each point of intersection find the equations of the line through that point and perpendicular to $\mathcal{L}$.

4) [6 marks] Find the set of values of x for which

$$\frac{(x-2)(x-1)}{(x+2)} < 1.$$

Turn over ...

5) [11 marks] Compute the integrals

$$i)\ 4\int \sin 5\theta \cos 9\theta\, d\theta \qquad \text{and} \qquad ii)\ \int \frac{4x^2 + 9x + 7}{(3+x)(x^2 + x + 2)}dx.$$

6) [9 marks] Compute enough terms of the Taylor expansion for the function

$$f(x) = \arctan(1 + x)$$

about $x = 0$ to approximate $f(x)$ by a cubic equation.