Mathematics for Actuarial Science (AS1051)

Coursework 1

This is an assessed coursework, and will count towards your final grade. Solutions should be handed in to the SEMS general office (C109) by the stated deadline. Late submissions will be penalised.

DEADLINE: Thursday 11/11/2010 at 15:00

1) *i)* Find the coefficient of x^2 in expansion of the expression [7 marks]

$$\left(2x^4 + \frac{5}{x^3}\right)^{11}$$

by considering the general term. Express your answer as a product of primes.

ii) Expand the expression

$$\left(1 + y^2 + y^3\right)^6$$

up to order y^7. Write down for this first the expansion of $(1+x)^6$ and subsequently replace x by $y^2 + y^3$ keeping terms to the appropriate order.

2) *i)* Use the appropriate indentities introduced in the lecture to show that [10 marks]

$$4\sin(\alpha+\beta)\sin(\alpha+\gamma)\sin(\beta+\gamma)+\sin(2\alpha+2\beta+2\gamma) = \sin(2\alpha)+\sin(2\beta)+\sin(2\gamma).$$

ii) Find the general solution for θ of

$$\cos(6\theta) + \cos(3\theta) + 4\cos^2\left(\frac{3}{2}\theta\right)\sin^2\left(\frac{3}{2}\theta\right) = 0.$$

Express your answer in multiples of π.

3) A circle with radius 2 and center located on the y-axis is inscribed into the parabola $y = x^2/2$. (This means the circle and the parabola have the same tangent at the points of intersection.) [10 marks]

i) Draw the corresponding figure.

ii) Determine the points of intersection, the center of the circle and the intersection of the circle with the y-axis.

4) *i)* Reexpress [16 marks]

$$f(x) = \frac{x^4 - x^2}{(x^2 - 2x + 4)(x^2 - 3x + 2)}$$

in terms of partial fractions.

ii) Differentiate the result from *i)* at $x = 0$.

iii) Compute the indefinite integral $\int f(x)dx$ using the result from *i)*.
Hint: You may use the fact that $\int \frac{1}{1+x^2}dx = \arctan x$.

5) For the two parametric functions [7 marks]

$$x(t) = t^3 + 3t + 1 \qquad \text{and} \qquad y(t) = \ln(3t^2 - 3)$$

find the function $p(t)$ such that

$$\frac{d^2y}{dx^2} = p(t)\frac{d^2y}{dt^2}\frac{d^2t}{dx^2}.$$