Mathematics for Actuarial Science (AS1051)

Coursework 1

This is an assessed coursework, and will count towards your final grade. Solutions should be handed in to the SEMS general office (C108) by the stated deadline. Late submissions will be penalised.

DEADLINE: Thursday 10/11/2011 at 15:00

1) *i)* Solve the following set of equations [10 marks]

$$\begin{aligned} x + y - 2z &= 12, \\ 2x - y + z &= -3, \\ w - x + 3y + z &= 9, \\ w + 5x + y + 2z &= 7, \end{aligned}$$

for w, x, y and z.

ii) Solve the equation

$$\log_3(x) - \log_9(|x - 12|) = 0$$

for x.

2) *i)* Use the appropriate identities introduced in the lecture to show that [10 marks]

$$\frac{\cos(2\theta - 2\phi)}{\cos(\theta - \phi) + \sin(\theta - \phi)} = (\cos\theta - \sin\theta)\cos\phi + \cos\theta\sin\phi + \sin\theta\sin\phi.$$

ii) Find the general solution for θ of

$$2\cos 2\theta + 4\sin\theta\cos\theta = \sqrt{2},$$

in the range $2\pi \leq \theta \leq 4\pi$. Express your answer in multiples of π.

3) A line $\mathcal{L}_1$ passess through the point (1,1) with gradient 3 and a second line $\mathcal{L}_2$ passess through the points (5,-9) and (50,0). [7 marks]

i) Find the equations for the two lines $\mathcal{L}_1$ and $\mathcal{L}_2$.

ii) Determine the equation for the line $\mathcal{L}_2^{\perp}$ perpendicular to $\mathcal{L}_2$ at (0,-10) and find the point of intersection between $\mathcal{L}_1$ and $\mathcal{L}_2^{\perp}$.

4) *i*) Reexpress [16 marks]

$$f(x) = \frac{2x^5 + x + 4}{(x-1)\,(x^2 - 3x + 2)\,(x^2 + x - 2)}$$

in terms of partial fractions.

ii) Differentiate the result from *i*) at $x = 0$ and $x = -1$.

iii) Compute the indefinite integral $\int f(x)dx$ using the result from *i*).

5) Using the identity [7 marks]

$$(1+x)^n(1-x)^n = (1-x^2)^n$$

deduce for $2k \leq n$ that

$$\sum_{l=0}^{2k} = (-1)^{k-l} \begin{pmatrix} n \\ l \end{pmatrix} \begin{pmatrix} n \\ 2k-l \end{pmatrix} = \begin{pmatrix} n \\ k \end{pmatrix}.$$