Mathematics for Actuarial Science 2

1. The sum of the first two terms of an arithmetic series is 47, and the thirtieth term of this series is -62. Find

   (a) the first term of the series and the common difference,

   (b) the sum of the first 60 terms of the series.

2. The first term of an arithmetic series is -13 and the last is 99. The sum of the series is 1419. Find the number of terms and the common difference. Find also the sum of all the positive terms in the series.

3. Find the sum of squares of the first n odd integers.

4. The function f is defined for all x except $x = 2$ by $f(x) = \frac{3x+1}{x-2}$.

   (a) The inverse function is denoted by f^{-1}. Obtain an expression for $f^{-1}(x)$.

   (b) Obtain an expression for $f \circ f(x)$. What is the domain of $f \circ f$?

5. Functions f and g are defined for all $x \in \mathbb{R}$ by

$$f(x) = 4 - x \qquad g(x) = 3x^2.$$

   (a) Find the range of g.

   (b) Solve $g \circ f(x) = 48$.

   (c) Sketch the graphs of $y = |f(x)|$ and $y = g(x)$ on the same axes. Find the values of x for which (i) $|f(x)| = 2$ and (ii) $|f(x)| = g(x)$.

6. For each of the following functions, find the largest domain and corresponding range.

$$(i)\ \ f(x) = x^2 - 3x + 2 \quad (ii)\ \ g(x) = \sqrt{x^2 - 3x + 2} \quad (iii)\ \ h(x) = |x^2 - 3x + 2|.$$

   Sketch the graphs of f and h.

7. Solve (i) $|3x - 2| < 4$ and (ii) $|x + 1| < |x - 2|$.

8. Find, in radians, all the values of θ between $-\pi$ and π satisfying the equation

$$8\cos^2\theta + 2\sin\theta = 7.$$

   Give your answers to three decimal places.

9. Solve the equation $\sin 2x = \cos^2 x$ for $0 \leq x \leq 360°$, giving your answer in degrees to one decimal place.

10. Solve the equation $\cos 2x = 2\sin^2 x$, giving the general solution in radians as a multiple of π.

11. Express $\cos 4x$ in terms of $\cos 2x$. Hence, or otherwise, express $\cos 4x$ in terms of $\sin x$. If $\cos 4x = a$, express $\sin x$ in terms of a.

12. Use a suitable trigonometric identity to solve the equation

$$\tan^2 x = 2\sec x + 2$$

for x, in radians, in the interval $-\pi < x < \pi$, giving your answers to 3 decimal places.

13. Given that $4\cos\theta + 3\sin\theta = R\cos(\theta - \alpha)$ find the value of R and the value of α, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

    (a) Hence find all values of θ between 0 and 2π satisfying

        i. $4\cos\theta + 3\sin\theta = 2$,

        ii. $4\cos 2\theta + 3\sin 2\theta = 5\cos\theta$.

    (b) Find the greatest and least values of the expression

$$\frac{1}{4\cos\theta + 3\sin\theta + 6}$$

    and give the corresponding values of θ between 0 and 2π.

14. Obtain an expression for y in terms of x if

$$\ln(x^3) + \ln(xy) - \ln(y^2) = 0.$$

15. If x satisfies $3^x.4^{2x+1} = 6^{x+2}$, show that $x = \frac{\ln 9}{\ln 8}$.

16. Find x satisfying $e^{2x} - 5e^x + 6 = 0$.

17. If $\ln a = p$ and $\ln b = q$, express

    (a) $\ln\left(\frac{a}{\sqrt{b}}\right)$ in terms of p and q,

    (b) $\exp(2p - 3q)$ in terms of a and b.

18. Solve the equations

    (a) $\log_3(9x - 2) = 2 + 2\log_3 x$,

    (b) $\log_x 8 + \log_8 x = \frac{13}{6}$.

19. Find the set of values of x for which $2x(x + 3) > (x + 2)(x - 3)$.

20. Solve the inequality $\frac{x-3}{2-x} > 1$.

21. Solve the inequalities

    (a) $2x^3 - 5x^2 - 4x + 3 > 0$,

    (b) $2e^{3u} - 5e^{2u} - 4e^u + 3 > 0$.