Mathematics for Actuarial Science 3

1. Solve, if possible, the system of equations

$$2x + 3y = 5 \qquad \text{and} \qquad 3x - 6y = 11.$$

2. Solve, if possible, the system of equations

$$\begin{aligned} x \quad -2y \quad -z \quad &= 6 \\ 3x \quad -6y \quad -5z \quad &= 3 \\ 2x \quad -y \quad +z \quad &= 0. \end{aligned}$$

3. Solve, if possible, the system of equations

$$\begin{aligned} x \quad +2y \quad +3z \quad &= 1 \\ -2x \quad -y \quad +2z \quad &= 2 \\ 3x \quad +3y \quad +z \quad &= 3. \end{aligned}$$

4. $(*)$ Find all possible solutions to the equations

$$\begin{aligned} x \quad +y \quad +3z \quad &= 1 \\ 3x \quad -y \quad +2z \quad &= 2 \\ 8x \quad -4y \quad +3z \quad &= 5. \end{aligned}$$

5. Find the equation of the line through $(9, -1)$ perpendicular to the line $2x + 3y = -11$. Calculate the coordinates of the point where these two lines meet.

6. The straight line through $P(2, 1)$ and $Q(k, 11)$ has gradient $-\frac{5}{12}$. Find an equation for the line in terms of x and y, determine the value of k, and calculate the distance between P and Q.

7. Find the centres and radii of the circles

$$x^2 + y^2 + 8x + 10y - 4 = 0 \qquad \text{and} \qquad x^2 + y^2 - 2x - 4 = 0.$$

   Find also the distance between their centres and hence:

   (a) show that the circles intersect at right angles;

   (b) find the length of their common chord (i.e. the distance between their points of intersection).

8. The equation of a circle with centre C is

$$(x - 3)^2 + (y - 4)^2 = 9$$

   and O is the origin. The line joining O and C can be extended to meet the circle at P. Find the coordinates of P and show that the equation of the tangent to the circle at P is $3x + 4y = 40$.

9. Find the focus, directrix and axis of each of the following parabolas, and sketch the corresponding curves.

   (a) $y = x^2 - 2x + 3$.

   (b) $x = y^2 + 2y - 4$.

   (c) $y = -4x^2 + 3x$.

10. Find the equation of the parabola with focus at $(6, 0)$ and directrix $x = 0$.

11. Find the centre, foci, and lengths of major/minor axes of the ellipses:

   (a) $\frac{x^2}{9} + \frac{y^2}{4} = 1$,

   (b) $4x^2 + 9y^2 - 18y = 27$,

   (c) $4x^2 + y^2 - 6y + 5 = 0$.

12. Find an equation for the ellipse with

   (a) Foci at $(\pm 1, 0)$ and major axis of length 6,

   (b) Focus at $(1, 1)$, centre at $(1, 3)$, and major axis of length 10,

   (c) Foci at $(-4, -1)$ and $(4, -1)$) and major axis of length 10.

13. Find the centre, foci, and asymptotes of the hyperbolas

   (a) $x^2 - y^2 = 1$,

   (b) $\frac{x^2}{9} - \frac{y^2}{16} = 1$,

   (c) $(*)$ $4x^2 - 8x - y^2 + 6y - 1 = 0$.