Mathematics for Actuarial Science 7

1. Calculate

$$\text{(a)} \int \frac{1}{4\sin x - 3\cos x - 5} dx \qquad \text{(b)} \int \frac{1}{3\cos x + 4\sin x} dx.$$

2. Using half angle identities show that

$$\int \sec x \, dx = \ln|\sec x + \tan x| + C.$$

3. Show that

$$\cosh^2 x + \sinh^2 x = \cosh 2x.$$

4. Show that

$$\cosh(x+y) = \cosh x \cosh y + \sinh x \sinh y.$$

5. Find an expression for $\cosh 3x$ in terms of $\cosh x$.

6. Show that

$$(\cosh x + \sinh x)^n = \cosh nx + \sinh nx.$$

(Hint: Do not expand the bracketed term!)

7. Solve

$$5\cosh x + \sinh x = 7.$$

8. Solve

$$3\text{sech}^2 x + 4\tanh x + 1 = 0.$$

9. Differentiate

$$\text{(a)} \ \tanh^4 3x \qquad \text{(b)} \ \text{sech}\left(\frac{1-x}{1+x}\right).$$

10. Differentiate

$$\text{(a)} \ \sinh^{-1}(\tan x) \qquad \text{(b)} \ \tanh^{-1}(\sin x).$$

11. Differentiate

$$\cosh^{-1}(\sec x)$$

and hence show (using the ln form of a suitable inverse hyperbolic function) that for $0 \leq x < \frac{\pi}{2}$ we have

$$\int \sec x \, dx = \ln(\sec x + \tan x) + C.$$

Why do we have modulus signs in the corresponding formula in Question 2?

12. Calculate

$$\int \frac{1}{\sqrt{9x^2 + 18x + 2}} dx.$$