Mathematics for Actuarial Science 8

1. Determine the following limits:

   (a)
   $$\lim_{x\to -2} \frac{x^2+6x+8}{x+2}.$$

   (b)
   $$\lim_{x\to 2} \frac{x-2}{\sqrt{x^2-4}}.$$

   (c)
   $$\lim_{x\to 0} \frac{\tan x}{x}.$$

   (d)
   $$\lim_{x\to 0} \frac{1-\cos(ax)}{x^2}.$$

   (e)
   $$\lim_{x\to 0} \frac{(x^2+2)\sin(3x)}{x}.$$

   (f)
   $$\lim_{x\to \infty} \frac{x^2+x+1}{x^2-x+1}.$$

   (g)
   $$\lim_{x\to 0} \frac{\tanh x}{x}.$$

   (h)
   $$\lim_{x\to \infty} \frac{\sqrt{x+\sqrt{x+1}}}{\sqrt{x+\sqrt{1+\sqrt{x}}}}.$$

2. Give the Maclaurin series for each of the following functions:

   (a) $2\sin x\cos x$.

   (b) $\ln\sqrt{\left(\frac{1-x}{1+x}\right)}$.

   (c) $\ln[(1+x)(1-2x)^2]$.

   (d) $\sin^3 x$.

3. Give the Taylor series for $\cos x$ about $\frac{\pi}{2}$ to three terms.

4. If $y=\tan(e^x-1)$, prove that
   $$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \frac{\mathrm{d}y}{\mathrm{d}x}(1+2e^x y).$$
   Obtain the Maclaurin expansion of y up to and including the x^4 term.

5. Use Leibnitz' theorem to compute the fifth derivative of

$$\text{(a) } x^3 \cos x \qquad \text{(b) } x^2 \ln x.$$

6. If $y = \frac{x+1}{x^2+2x+3}$ show that

$$(x^2 + 2x + 3)\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + 4(x + 1)\frac{\mathrm{d}y}{\mathrm{d}x} + 2y = 0.$$

Use Leibnitz' theorem to establish that

$$(x^2 + 2x + 3)\frac{\mathrm{d}^{n+2} y}{\mathrm{d}x^{n+2}} + 2(n + 2)(x + 1)\frac{\mathrm{d}^{n+1} y}{\mathrm{d}x^{n+1}} + (n + 1)(n + 2)\frac{\mathrm{d}^n y}{\mathrm{d}x^n} = 0.$$