AS1051

CITY UNIVERSITY
London

BSc Honours Degree in Actuarial Science

Part I Examination

Mathematics for Actuarial Science 2

2008

Time allowed: 2 hours

Full marks may be obtained for correct answers to
ALL of the SIX questions in Section A
and
TWO of the THREE questions in Section B.
If more than TWO questions from Section B are answered,
the best TWO marks will be credited.

Turn over . . .

Section A

Answer **all** questions from this section. Each question carries 8 marks.

1. Find the centre, foci, and asymptotes of the hyperbola

$$4x^2 - 8x - y^2 + 6y - 1 = 0.$$

[8]

2. Prove by induction that the sum of the first n odd natural numbers is n^2.

[8]

3. (a) Find the general solution of $2u_{n+1} = 6u_n + 2$, $n \geq 0$.

   (b) Find the solution of $u_{n+2} - 3u_{n+1} - 4u_n = 1$, $n \geq 0$ with $u_0 = \frac{2}{3}$ and $u_1 = \frac{2}{3}$.

[8]

4. Let a be a real number and M the matrix given by

$$\begin{pmatrix} 1 & 1 & 0 \\ 3 & a & 2 \\ 2 & 1 & a \end{pmatrix}$$

   (a) Find the values of a for which M is not invertible.

   (b) Find the inverse of M for $a = 0$.

[8]

5. Find all four complex solutions to

$$z^4 = \frac{1}{2} + i\frac{\sqrt{3}}{2}.$$

[8]

Turn over ...

6. We define the relation ρ on the set of complex numbers $\mathbb{C}$ by

$$z' \; \rho \; z \Leftrightarrow \exists R \in \mathbb{C} \;, \; R \neq 0 \text{ such that } z' = Rz \,.$$

(a) Give the definition of an equivalence relation on a set S.

(b) Show that ρ is an equivalence relation on $\mathbb{C}$.

[8]

Turn over . . .

Section B

Answer **two** questions from this section. Each question carries 26 marks.

7. (a) Find in terms of π the general solutions to

$$\sin 5\theta + \sin\theta = \sin 3\theta.$$

[10]

(b) Express $\cos 4x$ in terms of $\cos 2x$. Hence, or otherwise, express $\cos 4x$ in terms of $\sin x$. **[6]**

(c) Write down the definition of $\sin^{-1} x$, giving the domain and the range of the function. **[3]**

(d) Find the greatest and least values of

$$\frac{1}{4\cos x + 3\sin x + 8}.$$

[7]

8. For $n \in \mathbb{N}$, define

$$u_n = \frac{n(n+1)(2n+1)}{6}.$$

(a) Show that for all $j \geq 1$,

$$u_j - u_{j-1} = j^2.$$

[7]

(b) The area under the curve of $x \mapsto x^2$ between $x = 0$ and $x = 1$ can be successively approximated by

$$A_n = \frac{1}{n}\sum_{j=1}^{n} \frac{j^2}{n^2}.$$

Using (a), compute A_n for $n \geq 1$. **[10]**

(c) Deduce that

$$\lim_{n\to\infty} A_n = \int_0^1 x^2 dx$$

by computing the common value of these two expressions. **[9]**

Turn over ...

9. (a) Write the following system of equations in matrix form:

$$\begin{aligned} x - 3y - 2z &= 2, \\ -2x + y - z &= 1, \\ -x + 2y + a^2 z &= a, \end{aligned}$$

where a is a constant. By considering the rank of the resulting matrix and augmented matrix, say how many solutions there are for this system of equations for the three cases $a = -1$, 0 and 1. Do *not* calculate the solution to the equations when possible. **[13]**

(b) Using row manipulation methods, find the inverse of the following matrix:

$$\begin{pmatrix} 1 & -2 & -1 \\ -3 & 1 & 2 \\ -2 & -1 & 0 \end{pmatrix}.$$

[13]

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