AS1051

CITY UNIVERSITY
London

BSc Honours Degree in Actuarial Science

PART I EXAMINATION

Mathematics for Actuarial Science 2

2009

Time allowed: 2 hours

Full marks may be obtained for correct answers to
ALL of the SIX questions in Section A
and
TWO of the THREE questions in Section B.
If more than TWO questions from Section B are answered,
the best TWO marks will be credited.

Turn over . . .

Section A

Answer **all** questions from this section. Each question carries 8 marks.

1. Find an equation for the ellipse with focus at $(1,6)$, centre at $(1,8)$ and major axis of length 10. [**8**]

2. Prove by induction that $2^{n+2} + 3^{2n+1}$ is divisible by 7. [**8**]

3. We define the relation ρ on the set of complex numbers $\mathbb{C}$ by

$$z' \; \rho \; z \Leftrightarrow \exists n \in \mathbb{Z} \text{ such that } \arg z' - \arg z = 2n\pi\,,$$

where $\arg z$ means argument of z.

   (a) Give the definition of an equivalence relation on a set S.

   (b) Show that ρ is an equivalence relation on $\mathbb{C}$.

[**8**]

4. (a) Find the general solution of $3u_{n+1} = 6u_n + 3$, $n \geq 0$.

   (b) Find the solution of $u_{n+2} - u_{n+1} - 6u_n = 6$, $n \geq 0$ with $u_0 = 1$ and $u_1 = 5$.

[**8**]

5. Let a be a real number and M the matrix given by

$$\begin{pmatrix} a & 1 & 0 \\ 3 & 1 & 2 \\ 2 & 1 & a \end{pmatrix}$$

   (a) Find the values of a for which M is not invertible.

   (b) Find the inverse of M for $a = 0$.

[**8**]

6. Let $n \geq 1$ be an integer. Let $\omega = \cos \frac{2\pi}{n} + i \sin \frac{2\pi}{n}$.

(a) Show that $\omega^n = 1$.

(b) Let $n \geq 2$. Using the formula for the sum of the first n terms of a geometric sequence, deduce that

$$1 + \omega + \omega^2 + \cdots + \omega^{n-1} = 0\,.$$

[**8**]

Section B

Answer **two** questions from this section. Each question carries 26 marks.

7. (a) Find the general solution of the equation

$$2 \sin 2\theta = \sqrt{3} \tan 2\theta.$$

[**9**]

(b) If $2x + y = \frac{\pi}{4}$ show that

$$\tan y = \frac{1 - 2\tan x - \tan^2 x}{1 + 2\tan x - \tan^2 x}.$$

Hence deduce that $\tan \frac{\pi}{8}$ is a root of $t^2 + 2t - 1$, and that its value is $\sqrt{2} - 1$. [**8**]

(c) Write down an expression for $\cosh^{-1} x$ in terms of ln, and state for which values the function is defined. [**3**]

(d) Solve the equation

$$\cosh^2 x + \sinh^2 x = 4.$$

[**6**]

8. (a) Prove by induction that for all $n \geq 1$,

$$\sum_{j=1}^{n} j^3 = \left(\sum_{j=1}^{n} j \right)^2 .$$

Hence deduce an explicit expression for $\sum_{j=1}^{n} j^3$ in terms of n. [**10**]

(b) The area under the curve of $x \mapsto x^3$ between $x = 0$ and $x = 1$ can be successively approximated by

$$A_n = \frac{1}{n} \sum_{j=1}^{n} \frac{j^3}{n^3} .$$

Using (a), compute A_n for $n \geq 1$. [**8**]

(c) Deduce that

$$\lim_{n\to\infty} A_n = \int_0^1 x^3 dx$$

by computing the common value of these two expressions. **[8]**

9. Consider a function f of two real variables x, y. What is the condition for it to have stationary points? What test can one use to determine whether a stationary point is a maximum, minimum or saddle point? **[6]**

Let

$$f(x,y) = x^3 + y^3 + 3x^2 - 3y^2 - 8 \ , \ \forall x, y \in \mathbb{R} \,.$$

Determine its stationary points together with their nature.

[20]

Internal Examiners:	Dr A. G. Cox
	Dr V. Caudrelier
External Examiners:	Professor J. Billingham
	Professor E. Corrigan