AS1051

CITY UNIVERSITY

London

BSc Honours Degree in Actuarial Science

Part I Examination

Mathematics for Actuarial Science 1

2010

Time allowed: 2 hours

Full marks may be obtained for correct answers to
ALL of the SIX questions in Section A
and
TWO of the THREE questions in Section B.
If more than TWO questions from Section B are answered,
the best TWO marks will be credited.

Section A

Answer **all** questions from this section. Each question carries 8 marks.

1. Solve the equation

$$\sinh^2 x + 10 = 6\cosh x$$

giving your answer in logarithmic form. **[8]**

2. (a) Give the definition of the Taylor series of a function f about c. **[2]**

(b) Calculate from first principles the Taylor series of the function

$$f(x) = x^2 \ln(x)$$

about 1 up to the term in $(x-1)^3$. **[6]**

3. Calculate

(a)

$$\int x^2 e^{-2x}\, dx.$$

[4]

(b)

$$\int \frac{3x}{\sqrt{3x+2}}\, dx.$$

[4]

4. Evaluate each of the following limits:

(a)

$$\lim_{x\to 0} \left(\frac{\cosh x - e^x \cos x}{\sin 3x} \right);$$

[4]

(b)

$$\lim_{x\to\infty} \left(\frac{5x^2 + 3x - 7}{2 - 3x^2} \right).$$

[4]

Turn over ...

5. If

$$I_n = \int_0^{\pi} x^n \cos x \, dx,$$

show that

$$I_n = -n\pi^{n-1} - n(n-1)I_{n-2}.$$

[**4**]

Hence evaluate

$$\int_0^{\pi} x^6 \cos x \, dx.$$

[**4**]

6. Find the general solution to the equation

$$\frac{dy}{dx} + 2y\tan(x) = \sin x$$

[**8**]

Turn over ...

Section B

Answer **two** questions from this section. Each question carries 26 marks.

7. (a) Find the general solution of the equation

$$\sin 5\theta = \cos 7\theta.$$

[8]

(b) Write down the definition of $\tan^{-1} x$, giving the domain and range of the function. **[3]**

(c) Suppose that a and b are such that

$$-\frac{\pi}{2} < \tan^{-1} a + \tan^{-1} b < \frac{\pi}{2}.$$

Show that

$$\tan^{-1} a + \tan^{-1} b = \tan^{-1}\left(\frac{a+b}{1-ab}\right).$$

[6]

(d) Why is the condition on a and b in the last part necessary? **[2]**

(e) By differentiating the equation

$$\frac{x}{a} = \tan(y)$$

with respect to x show that

$$\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C.$$

[7]

8. A function $f(x, y)$, has stationary points where both $\frac{\partial f}{\partial x} = 0$ and $\frac{\partial f}{\partial y} = 0$ simultaneously. What test should be used for identifying whether a stationary point is a maximum, a minimum or a saddle point? **[4]**

For the function

$$f(x, y) = x^3 + y^3 - 2(x^2 + y^2) + 3xy$$

find all the stationary points, and identify their types. **[22]**

Turn over . . .

9. (a) Find the general solution of

$$y'' - 2y' + 5y = 0$$

and write the solution in a form that includes $\cos(bx)$ and $\sin(bx)$ where b is a value that should be determined. **[7]**

(b) Find the solution of

$$y'' - 2y' + y = e^x$$

subject to $y(0) = 1$, $\frac{dy(0)}{dx} = 0$ **[19]**

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