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AS1051

CITY UNIVERSITY

London

BSc Honours Degree in Actuarial Science

Part I Examination

Mathematics for Actuarial Science 2

2010

Time allowed: 2 hours

Full marks may be obtained for correct answers to
ALL of the SIX questions in Section A
and
TWO of the THREE questions in Section B.
If more than TWO questions from Section B are answered,
the best TWO marks will be credited.

Section A

Answer **all** questions from this section. Each question carries 8 marks.

1. Find the centre, foci, and asymptotes of the hyperbola

$$4y^2 - 16y - 9x^2 - 18x - 29 = 0.$$

[8]

2. Prove by induction that every set containing $n \geq 0$ elements has exactly 2^n subsets. **[8]**

3. (a) Find the general solution to $A_{n+1} = 4A_n + n + 2$. **[4]**

   (b) Find the general solution to $A_{n+2} + 3A_{n+1} - 4A_n = 5^n$ **[4]**

4. (a) Calculate the inverse of

$$\begin{pmatrix} 3 & 4 \\ 2 & 6 \end{pmatrix}.$$

[4]

   (b) Calculate the determinant of

$$\begin{pmatrix} 1 & 0 & 2 & 1 \\ 0 & 2 & 0 & 1 \\ 0 & 0 & 1 & 3 \\ 1 & 0 & 3 & 0 \end{pmatrix}.$$

[4]

5. Find all four complex solutions to

$$z^4 = i.$$

[8]

6. State the definition of:

   (a) a reflexive relation.

   (b) a symmetric relation.

   [4]

   Determine if the relation on $A = \{r, s, t\}$ given by

   $$R = \{(r, s), (s, r), (t, r), (r, t)\}$$

   is

   (a) symmetric

   (b) reflexive

   Give reasons for your answers. **[4]**

Section B

Answer **two** questions from this section. Each question carries 26 marks.

7. (a) Using a suitable double angle identity show that

$$\cos(x) = \frac{1-t^2}{1+t^2}$$

where $t = \tan(\frac{x}{2})$. **[7]**

(b) If $t = \tan(\frac{x}{2})$ determine $\frac{\mathrm{d}x}{\mathrm{d}t}$ as a function of t. **[4]**

(c) Hence (or otherwise) calculate

$$\int \frac{dx}{4\cos x + 5}.$$

[7]

(d) Write down a version of the identity in (a) involving cosh and tanh, and verify it directly using the definitions of these functions. **[8]**

8. (a) Use an appropriate test, determine if the series

$$\sum_{n=0}^{\infty} n \left(\frac{1+\sqrt{5}}{6}\right)^n$$

converges. **[6]**

(b) Let

$$S_n = 1 + 5x + 9x^2 + \cdots + [1 + 4(n-1)]x^{n-1}.$$

Use the method of differences to show that

$$\lim_{n\to\infty} S_n = \frac{1+3x}{(1-x)^2}$$

for $|x| < 1$. **[20]**

9. (a) Using row manipulation methods, find the inverse of the following matrix:

$$\mathbf{M} = \begin{pmatrix} 1 & 2 & 1 \\ 3 & 1 & 1 \\ 2 & -1 & 1 \end{pmatrix}$$

[12]

(b) Write the following system of equations in matrix form

$$\begin{aligned} x - 3y + 4z &= 1 \\ -x + y - 7z &= 1 \\ x - 3y + a^2 z &= 2a \end{aligned}$$

where a is a constant. Using Gaussian elimination, bring the system into upper triangular form. Find the solutions when a = 1 and a = 0. **[14]**