

AS 1051 Mathematics for Actuarial Science
January progress test 2006

Time allowed: 90 minutes.

Full marks can be obtained by answering all six questions.
All necessary working must be shown.

1. Let f be the function defined by

$$f(x) = \ln \left(\frac{x+2}{x+1} \right).$$

- (a) Find the domain of this function.
(b) Solve the inequality

$$f(x) \geq 0.$$

2. If $2x + y = \frac{\pi}{4}$, show that

$$\tan y = \frac{1 - 2 \tan x - \tan^2 x}{1 + 2 \tan x - \tan^2 x}.$$

Hence deduce that $\tan \frac{\pi}{8}$ is a root of $t^2 + 2t - 1 = 0$, and that its value is $\sqrt{2} - 1$.

3. Calculate the following integrals.

$$(a) \int \sin 8\theta \cos 3\theta \, d\theta \quad (b) \int \sin \theta \sqrt{1 - \cos \theta} \, d\theta \quad (c) \int \frac{3}{\sqrt{x^2 - 2x - 3}} \, dx.$$

4. State and prove an identity involving $\coth^2 x$ and $\operatorname{cosech}^2 x$, and hence solve

$$3 \coth x + \operatorname{cosech}^2 x = 3.$$

5. Use a Maclaurin series to give the quadratic polynomial which approximates

$$\frac{e^x}{1+x}.$$

6. Prove by induction that

$$\sum_{r=1}^n (2r-1)^2 = \frac{1}{3}n(2n-1)(2n+1).$$

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