AS 1051 Mathematics for Actuarial Science
January progress test 2007

Time allowed: 90 minutes.

Full marks can be obtained by answering all six questions.
All necessary working must be shown.

1. Calculate

$$\int_1^3 \frac{1-4x+3x^2}{(1+x)^2(1+3x)}dx.$$

[7]

2. The function $f(\theta)$ is defined for all θ by

$$f(\theta) = 2(\sin\theta + 2\cos\theta)^2 + \cos 2\theta - 5.$$

   (a) Show that $f(\theta) = 4\cos 2\theta + 4\sin 2\theta$.

   (b) Find $R > 0$ and $0 < \alpha < \pi/2$ such that $f(\theta) = R\cos(2\theta - \alpha)$.

   (c) Hence find the greatest and least values of $f(\theta)$.

   (d) Solve the equation $2(\sin\theta + 2\cos\theta)^2 + \cos 2\theta = 9$.

[10]

3. The equations of two circles are given by

$$x^2 + y^2 - 4x - 6y + 4 = 0 \qquad \text{and} \qquad x^2 + y^2 - 16x - 12y + 75 = 0.$$

   (a) Find the centre and radius of each circle.

   (b) Find the equation of the straight line passing through the centres of the two circles.

   (c) Find the distance between the two centres, and hence show that the circles intersect.

   (d) Find the equation of the smallest circle which passes through the centres of the two given circles.

[7]

4. Calculate the following integrals.

$$\text{(a)} \int x^2\cos 2x dx \qquad \text{(b)} \int_0^1 \frac{2x}{\sqrt{2x+1}}dx \qquad \text{(c)} \int \frac{3}{x^2+2x+5}dx.$$

[11]

[Continued overleaf]

5. State and prove an identity relating $\tanh^2 x$ and $\text{sech}^2 x$. Hence solve

$$3\text{sech}^2 x + 4\tanh x + 1 = 0.$$

[9]

6. Determine the Maclaurin series for $\cosh(2x)$ up to the term involving x^6. Hence evaluate

$$\lim_{x \to 0} \left(\frac{\cosh(2x) - 1}{x^2} \right).$$

[6]

Examiner: Dr A. G. Cox