AS 1051 Mathematics for Actuarial Science
January 2008

Time allowed: 90 minutes.

Full marks can be obtained by answering all six questions.
All necessary working must be shown.

1. Let f be the function defined by

$$f(x) = \ln\left(\frac{x+3}{2x+1}\right).$$

   (a) Find the domain of this function.

   (b) Solve the inequality

$$f(x) \geq 0.$$

2. If $2x + y = \frac{\pi}{4}$, show that

$$\tan y = \frac{1 - 2\tan x - \tan^2 x}{1 + 2\tan x - \tan^2 x}.$$

   Hence deduce that $\tan\frac{\pi}{8}$ is a root of $t^2 + 2t - 1 = 0$, and that its value is $\sqrt{2} - 1$.

3. Calculate the following integrals.

$$\text{(a)} \int \sin 3\theta \cos 7\theta \, d\theta \qquad \text{(b)} \int e^x \sqrt{1 - e^{2x}} \, dx \qquad .$$

4. State and prove an identity involving $\coth^2 x$ and $\text{cosech}^2 x$, and hence solve

$$3\coth x + \text{cosech}^2 x = 3.$$

5. Use a Maclaurin series to give the quadratic polynomial which approximates

$$\frac{e^{2x}}{1-x}.$$

6. Prove by induction that

$$\sum_{r=1}^{n} (2r-1)^2 = \frac{1}{3} n(2n-1)(2n+1).$$

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