

AS 1051 Mathematics for Actuarial Science
January 2010

Time allowed: 90 minutes.

*Full marks can be obtained by answering all six questions.
All necessary working must be shown.*

1. Let f be the function defined by

$$f(x) = \ln \left(\frac{x+3}{2x+1} \right).$$

- (a) Find the domain of this function.
(b) Solve the inequality

$$f(x) \geq 0.$$

2. If $2x + y = \frac{\pi}{4}$, show that

$$\tan y = \frac{1 - 2 \tan x - \tan^2 x}{1 + 2 \tan x - \tan^2 x}.$$

Hence deduce that $\tan \frac{\pi}{8}$ is a root of $t^2 + 2t - 1 = 0$, and that its value is $\sqrt{2} - 1$.

3. Calculate the following integrals.

$$(a) \int \sin 3\theta \cos 7\theta d\theta \quad (b) \int e^x \sqrt{1 - e^{2x}} dx \quad .$$

4. State and prove an identity involving $\coth^2 x$ and $\operatorname{cosech}^2 x$, and hence solve

$$3 \coth x + \operatorname{cosech}^2 x = 3.$$

5. Use a Maclaurin series to give the quadratic polynomial which approximates

$$\frac{e^{2x}}{1-x}.$$

6. Prove by induction that

$$\sum_{r=1}^n (2r-1)^2 = \frac{1}{3}n(2n-1)(2n+1).$$

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