Mathematics for Actuarial Science: Answer sheet 1

Sheet 1

1. $x = \sqrt{\frac{b}{a-c^2d}}$, 2.

2. $(p^2+q)+2p\sqrt{q}$,
$(p^2+3pq)+(3p^2+q)\sqrt{q}$.

4. $32-240x+720x^2-1080x^3+810x^4-243x^5$.

5. $A = 44$, $B = 924$, $C = 12320$.

7. 1215.

10. $\frac{1}{3}$, 1.

11. $3x^2-3x-2=0$.

12. $k \le -6$ or $k \ge 2$, (a) -155,
(b) $x^2-14x-479=0$.

13. x^4+3x^3-2x+1.

14. $(x-4)(x^2-4x+13)$, 4.

15. $(x-2)(x-3)(2x+1)$.

16. $a=-1$, $b=4$.

17. $k=9$.

18. $-\frac{2}{(1+x)^2}+\frac{3}{1+3x}$.

19. $\frac{1}{1-2x}-\frac{2x}{1+x^2}$.

20. $(x^2+2\sqrt{3}x+4)(x^2-2\sqrt{3}x+4)$,
$\frac{x+2\sqrt{3}}{16\sqrt{3}(x^2+2\sqrt{3}x+4)}-\frac{x-2\sqrt{3}}{16\sqrt{3}(x^2-2\sqrt{3}x+4)}$.

21. $1-\frac{1}{x^2}-\frac{2}{3(x+1)}-\frac{1-2x}{3(x^2-x+1)}$.

Sheet 2

1. $a=25$, $d=-3$, (b) $S=-3810$.

2. $n=33$, $d=\frac{7}{2}$, $S=1450$.

3. $\frac{1}{3}n(4n^2-1)$.

4. (a) $f^{-1}(x)=\frac{2x+1}{x-3}$, $x \ne 3$.
(b) $f(f(x))=\frac{10x+1}{x+5}$, $x \ne -5$, $x \ne 2$.

5. (a) $x \ge 0$, (b) 0, 8,
(c) i. 2, 6, ii. 1, $-\frac{4}{3}$.

6. (a) $\mathbb{R}$, $y \ge -\frac{1}{4}$, (b) $x \le 1$ or $x \ge 2$,
$y \ge 0$, (c) $\mathbb{R}$, $y \ge 0$.

7. (a) $-\frac{2}{3} < x < 2$, (b) $x < \frac{1}{2}$.

8. 0.524, 2.618, 3.386, 6.034.

9. 26.6, 90, 206.6, 270.

10. $\left(n \pm \frac{1}{6}\right)\pi$.

11. $\cos 4x = 2\cos^2 2x - 1$
$= 8\sin^4 x - 8\sin^2 x + 1$,
$\sin x = \pm\frac{1}{2}\sqrt{2 \pm \sqrt{2(1+a)}}$.

12. 1.231, 3.142, 5.052.

13. $R=5$, $\alpha = 0.6435$. (a) i. 1.80, 5.77,
ii. 0.21, 0.64, 2.31, 4.40.
(b) max 1 when $\theta = 3.785$,
min $\frac{1}{11}$ when $\theta = 0.644$.

14. $y=x^4$.

16. $\ln 2$, $\ln 3$.

17. (a) $p-\frac{1}{2}q$, (b) a^2b^{-3}.

18. (a) $\frac{1}{3}$, $\frac{2}{3}$, (b) 4, $16\sqrt{2}$.

19. $x < -6$, $x > -1$.

20. $2 < x < \frac{5}{2}$.

21. (a) $-1 < x < \frac{1}{2}$ or $x > 3$,
(b) $u < -\ln 2$ or $u > \ln 3$.