Mathematics for Actuarial Science: Answer sheet 2

Sheet 3

1. $x = 3$, $y = -\frac{1}{3}$.
2. $x = -\frac{19}{2}$, $y = -\frac{23}{2}$, $z = \frac{15}{2}$.
3. No solution.
4. $x = \frac{3-5z}{4}$, $y = \frac{1-7z}{4}$ (or similar).
5. $2y = 3x - 29$, $(5, -7)$.
6. $12y + 5x = 22$, $k = -22$, 26.
7. $(-4, -5)$, $3\sqrt{5}$; $(1, 0)$, $\sqrt{5}$, $5\sqrt{2}$, (b) $3\sqrt{2}$.
8. $(\frac{24}{5}, \frac{32}{5})$.
9. (a) F: $(1, \frac{9}{4})$, D: $y = \frac{7}{4}$, A: $x = 1$.
   (b) F: $(-\frac{19}{4}, -1)$, D: $x = -\frac{21}{4}$, A: $y = -1$.
   (c) F: $(\frac{3}{8}, \frac{1}{2})$, D: $y = \frac{5}{8}$, A: $x = \frac{3}{8}$.
10. $y^2 = 12x - 36$.
11. (a) C: $(0, 0)$, F: $(\pm\sqrt{5}, 0)$, major: 6, minor: 4.
    (b) C: $(0, 1)$, F: $(\pm\sqrt{5}, 1)$, major: 6, minor: 4.
    (e) C: $(0, 3)$, F: $(0, 3 \pm \sqrt{3})$, major: 4, minor: 2.
12. (a) $\frac{x^2}{9} + \frac{y^2}{8} = 1$.
    (b) $\frac{(x-1)^2}{21} + \frac{(y-3)^2}{25} = 1$.
    (c) $\frac{x^2}{25} + \frac{(y+1)^2}{9} = 1$.
13. (a) C: $(0, 0)$, F: $(\pm\sqrt{2}, 0)$, A: $y = \pm x$.
    (b) C: $(0, 0)$, F: $(\pm 5, 0)$, A: $y = \pm\frac{4}{3}x$.
    (c) C: $(1, 3)$, F: $(1, 3 \pm \sqrt{5})$, A: $y = 2x + 1$ and $y = -2x + 5$.

Sheet 4

1. (a) $12x^2 - 4x - 2x^{-3}$.
   (b) $-3\sin(3x + 2)$.
   (c) $\frac{1-x^2}{(1+x^2)^2}$.
   (d) $1 + \ln x$.
   (e) $\frac{x}{\sqrt{1+x^2}}$.
   (f) $e^x \cos(x^2) - 2xe^x \sin(x^2)$.
   (g) $\frac{6}{(2x-1)^2} - \frac{2}{(x+2)^2}$.
   (h) $x^{e^x}(\frac{1}{x}e^x + \ln(x)e^x)$.
   (i) $-\sin(\tan(x^2))\sec^2(x^2)2x$.
2. (a) $3x^2 + 3y^2 + 6xy\frac{\mathrm{d}y}{\mathrm{d}x} - 2y - 2x\frac{\mathrm{d}y}{\mathrm{d}x} - 4x^{-5}y^{-3} - 3x^{-4}y^{-4}\frac{\mathrm{d}y}{\mathrm{d}x} = 0$.
   (b) $-\sin(x)\sin(y) + \cos(x)\cos(y)\frac{\mathrm{d}y}{\mathrm{d}x} = 0$.
   (c) $-\sin(y\tan(x))(\frac{\mathrm{d}y}{\mathrm{d}x}\tan(x) + y\sec^2(x)) = (xy)^{-1}(y + x\frac{\mathrm{d}y}{\mathrm{d}x})$.
   (d) $\frac{(1+2\cos(y)\frac{\mathrm{d}y}{\mathrm{d}x})(x-y)-(x+2\sin(y)+4)(1-\frac{\mathrm{d}y}{\mathrm{d}x})}{(x-y)^2} = -\mathrm{cosec}^2(y)\frac{\mathrm{d}y}{\mathrm{d}x}$.
   (e) $\tan(x^{-1}\frac{\mathrm{d}y}{\mathrm{d}x} - y\sec^2(x^{-1})x^{-2} + \sec(y^{-1}) - x\sec(y^{-1})\tan(y^{-1})y^{-2}\frac{\mathrm{d}y}{\mathrm{d}x} = e^x$.
3. (a) $\frac{21t^2}{2t+2}$
   (b) $\mathrm{cosec}^3(t)$
   (c) $\frac{(3t^2+1)(1+2t^2)}{2t}$
   (d) $e^{-2t^2}\frac{\cos(t)-2t\sin(t)}{2t\cos(t)-\sin(t)}$
   (e) $-12t\sec^2(4t)\ln(t)^2$.
4. 8.
5. $f'(x) = -\frac{x}{2} + \frac{1}{x}$, $\beta = \sqrt{2}$.