

Mathematics for Actuarial Science: Answer sheet 4

Sheet 7

1. (a) $\frac{1}{\tan \frac{x}{2} - 2} + C$, (b) $-\frac{1}{5} \ln |\tan \frac{x}{2} - 3| + \frac{1}{5} \ln |3 \tan \frac{x}{2} + 1| + C$.
5. $4 \cosh^3 x - 3 \cosh x$.
7. $\ln 2, -\ln 3$.
8. $\frac{1}{2} \ln \frac{1}{5}$.
9. (a) $12 \tanh^3(3x) \operatorname{sech}^2(3x)$, (b) $\frac{2}{(1+x)^2} \operatorname{sech}(\frac{1-x}{1+x}) \tanh(\frac{1-x}{1+x})$.
10. (a) $\sec x$, (b) $\sec x$.
11. $\sec x$. Because Q2 is not restricted to the case $0 \leq x < \frac{\pi}{2}$.
12. $\frac{1}{3} \cosh^{-1} \left(\frac{3x+3}{\sqrt{7}} \right) + C$.

Sheet 8

1. (a) 2, (b) 0, (c) 1, (d) $\frac{a^2}{2}$, (e) 6, (f) 1, (g) 0, (h) 1.
2. (a) $\frac{(-1)^n 2^{2n+1} x^{2n+1}}{(2n+1)!}$, (b) $\frac{-x^{2n+1}}{2n+1}$, (c) $\frac{x^n}{n} [(-1)^{n+1} - 2^{n+1}]$, (d) $\frac{(-1)^n}{4(2n+1)!} (3 - 3^{2n+1}) x^{2n+1}$.
3. $-(x - \frac{\pi}{2}) + \frac{1}{6}(x - \frac{\pi}{2})^3 - \frac{1}{120}(x - \frac{\pi}{2})^5$.
4. $x + \frac{1}{2}x^2 + \frac{1}{2}x^3 + \frac{13}{24}x^4$.
5. (a) $-x^3 \sin x + 15x^2 \cos x + 60x \sin x - 60 \cos x$, (b) $\frac{4}{x^3}$.