Mathematics for Actuarial Science: Answer sheet 5

Sheet 9

1. (a) (i) 1, 2, 3. (ii) $\emptyset$, $\{1\}$, $\{2\}$, $\{3\}$, $\{1,2\}$, $\{1,3\}$, $\{2,3\}$, $\{1,2,3\}$.
(b) (i) R, $\{G\}$, B. (ii) $\emptyset$, $\{R\}$, $\{\{G\}\}$, $\{B\}$, $\{R,\{G\}\}$, $\{R,B\}$, $\{\{G\},B\}$, $\{R,\{G\},B\}$.
(c) (i) R, $\{R\}$. (ii) $\emptyset$, $\{R\}$, $\{\{R\}\}$, $\{R,\{R\}\}$.

2. (a) T, (b) F, (c) F, (d) T, (e) T, (f) F.

3. (a) F, e.g $A=\{1\}$, $B=\{2\}$, $C=\{1,2\}$, (b) T, (c) F, could have $A=B=C$, (d) F, could have $A=B$, (e) T.

5. Various possible examples. It is best to split the sentence into the simplest components. We will give sample solutions.

   (a) Let m = "I will go out for a meal", f = "I will see a film", c = "I finish my coursework", and r = "run out of money". Then the sentence becomes

   $$(c \wedge (\neg r)) \longrightarrow (m \wedge f).$$

   (b) Let r = "I will cut the red wire", g = "I will cut the green wire", b = "I will cut the blue wire", e = "the bomb will explode", and s = "we will be saved". Then the sentence becomes

   $$(r \vee (g \wedge b)) \longrightarrow ((\neg e) \wedge s).$$

6. (a), T, (b) F, (c) F, (d) T, (e) T, (f) F, (g) T.

7. False when $p=F$, $q=F$, $r=F$.

9.
$$\neg\left(\{[(\exists x)(\forall t)p(x,t)] \wedge [(\forall x)(\exists t)p(x,t)]\} \longrightarrow [(\forall x)(\forall t)p(x,t)]\right)$$

10. (a) F, (b) F, (c) T, (d) T, (e) F.

Sheet 10

As they are proofs, answers for Sheet 10 are not given here.