Maths for Actuarial Science Coursework 1: Answers

1. Easiest way is to sketch curves and inspect. Algebraically, any method is OK provided all cases are considered. Please deduct marks for each time a case is left out. One way: Let
$$A = \frac{x+1}{x-2} \quad \text{and} \quad B = \frac{x-1}{x+2}.$$
Equation equivalent to $A^2 > B^2$. Now multiply through and simplify.
The denominator equals $(x-2)^2(x+2)^2$ which is positive except at $x = \pm 2$ where the original equation is not defined.
The numerator simplifies to $x(x^2+2)$ and we require this to be positive.
As $x^2 + 2 > 0$, the solution is $x > 0$ with $x \neq 2$. **[Total: 10]**

2. (i) Centre of ellipse is midway between foci, so centre is $(3, 6)$. **[1]**
Major axis of length $2a = 6$, so $a = 3$. **[1]**
Distance from foci to centre is $ae = 1$, so $e = \frac{1}{3}$. **[1]**
Therefore $b^2 = a^2(1-e^2) = 8$. **[1]**
The equation is thus
$$\frac{(y-6)^2}{9} + \frac{(x-2)^2}{8} = 1.$$
[1]
(ii) Any tangent line at a point X is perpendicular to a radius passing through X. Consider the right-angled triangle OXP where O is the centre of the circle. This has hypotenuse of length $|OP|$ and one side of length the circle radius, so the length of the third side (by Pythagoras) is independent of the choice of X. (No marks for doing an example only. A picture is sufficient if it is explained properly.) **[5]**
[Total: 10]

3. (i) Verification of identity. **[5]**
(ii) We have
$$\sin 5\theta + \sin\theta = 2\sin 3\theta \cos 2\theta = \sin 3\theta$$
and so we must solve
$$\sin 3\theta(2\cos 2\theta - 1) = 0.$$
[2]
For $\sin 3\theta = 0$ we have general solution
$$\theta = \frac{n\pi}{3}$$
with $n \in \mathbb{Z}$, and for $\cos 2\theta = \frac{1}{2}$ we have
$$\theta = \left(n \pm \frac{1}{6}\right)\pi$$
with $n \in \mathbb{Z}$. (Please deduct a mark for approximate answers, and for not stating the domain of n.) **[3]**
[Total: 10]

4. (i)(a)

$$3(1+3x)(1+2\ln(1+3x)).$$

[2]

(b)

$$6x^2\tan(x^3+1)\sec^2(x^3+1).$$

[2]

(ii) $\frac{\mathrm{d}x}{\mathrm{d}t}=2t+1$ and $\frac{\mathrm{d}y}{\mathrm{d}t}=1/t$ so

$$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{t(2t+1)}.$$

[3]

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)=\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{t(2t+1)}\right)\frac{\mathrm{d}t}{\mathrm{d}x}=\frac{-(4t+1)}{t^2(2t+1)^3}.$$

[3]
[Total: 10]

5. (a)

$$\int\frac{x+2}{1-4x^2}\,dx=\int\frac{5}{4(1-2x)}+\frac{3}{4(1+2x)}\,dx=-\frac{5}{8}\ln(1-2x)+\frac{3}{8}\ln(1+2x)+C.$$

Please deduct a mark for a missing constant of integration. **[5]**

(b) Let $u=\sqrt{2x+1}$, so $x=\frac{u^2-1}{2}$ and $\frac{\mathrm{d}x}{\mathrm{d}u}=u$.

$$\int_0^1\frac{2x}{\sqrt{2x+1}}\,dx=\int_1^{\sqrt{3}}\frac{u^2-1}{u}u\,du=\left[\frac{u^3}{3}-u\right]_1^{\sqrt{3}}=\frac{2}{3}.$$

[5]
[Total: 10]