Dynamical Systems II

Coursework

Hand in the complete solutions to all three questions in the SEMS general office (C109).

DEADLINE: Thursday 15/11/2012 at 16:00

1) [25 marks] Consider the dynamical system

$$\begin{aligned}\dot{x}_1 &= 2\cos x_1 - \cos x_2,\\ \dot{x}_2 &= 2\cos x_2 - \cos x_1,\end{aligned}$$

- (i) Determine all fixed points of the system.
- (ii) Classify the nature of the four fixed points in the region

$$\mathcal{D} = \{(x_1, x_2) : -\pi \leq x_1 \leq \pi, -\pi \leq x_2 \leq \pi\} \ .$$

- (iii) Use the linearization theorem to sketch the local phase portraits around the four fixed points determined in (ii).
- (iv) Determine the isocline $dx_2/dx_1 = 1$.
- (v) Assemble the information from (ii), (iii), (iv) and use it to sketch the phase portrait for the above system in $\mathcal{D}$. Provide reasoning for drawing your trajectories.

2) [5 marks] Consider the dynamical system

$$\begin{aligned}\dot{x}_1 &= -x_1 + 4x_2,\\ \dot{x}_2 &= -x_1 - x_2^3.\end{aligned}$$

Determine λ such that $V(x_1, x_2) = x_1^2 + \lambda x_2^2$ becomes a strong Lyapunov function for the above system in $\mathbb{R}^2$.

3) [30 marks] Consider the dynamical system

$$\begin{aligned}\dot{x}_1 &= x_1(1 - 4x_1^2 - x_2^2) - \frac{1}{2}x_2(1 + x_1),\\ \dot{x}_2 &= x_2(1 - 4x_1^2 - x_2^2) + 2x_1(1 + x_1).\end{aligned}$$

(i) Classify the nature of the fixed point at the origin.

(ii) Use the Poincaré-Bendixson theorem to argue that there exists at least one limit cycle in the annular region

$$\mathcal{D} = \left\{ (r, \vartheta) : \frac{1}{8} \leq r \leq 2 \right\} .$$

(iii) By using the function $V(x_1, x_2) = \left(1 - 4x_1^2 - x_2^2\right)^2$ consider $\dot{V}$ to argue that the ellipse $4x_1^2 + x_2^2 = 1$ is an asymptotically stable limit cycle.

(iv) Determine all α and ω limit sets for the above system.