Dynamical Systems II

Coursework 2

Hand in the complete solutions to all two questions in the SEMS general office (C109).

DEADLINE: Thursday 13/12/2012 at 16:00

1) [20 marks]

(i) Provide a criterium that can be used to decide whether a system is Hamiltonian or not. Employ the criterium to determine the value of α, β and γ such that the system

$$\begin{aligned}\dot{x}_1 &= \alpha\beta x_1^2 x_2^2 \exp[(\alpha+\gamma)x_1^3] + 4x_1 + x_2 \\ \dot{x}_2 &= \alpha^2 x_1^4 x_2^3 \exp[(\alpha+\gamma)x_1^3] + (a+\beta)x_1 x_2^3 \exp[(\alpha+\gamma)x_1^3] + 2\gamma x_2\end{aligned}$$

becomes a Hamiltonian system.

(ii) Construct the Hamiltonian for all your solutions in (i).

(iii) Derive the Hamiltonian function for the dynamical system

$$\begin{aligned}\dot{x}_1 &= x_2 \\ \dot{x}_2 &= 4x_1^3 - 2\cosh x_1 \sinh x_1\end{aligned}$$

and confirm that it is a potential system. Fix the potential in such way that its value at zero is zero.

(iv) Use the fact that the system in (iii) is a potential system to sketch the corresponding phase portrait.

2) [20 marks] Consider the following difference equation

$$x_{n+1} = F(x_n) = 3x_n - 3\lambda x_n + \lambda x_n^2 \qquad \text{for } \lambda \in \mathbb{R}^+.$$

λ is taken to be the bifurcation parameter.

(i) Depending on the values of λ, determine the nature of the fixed points and their stability.

(ii) Determine the equation that governs the existence of 2-cycles. Compute the solution of that equation and use it to argue for which values of λ the 2-cycles exist.

(iii) Determine the domain of stability for the 2-cycle.