Dynamical Systems Exercises 1

1) Determine the fixed points of the following dynamical systems in the plane:

i)

$$\dot{x}_1 = \alpha x_1 - \beta x_1 x_2 \qquad \dot{x}_2 = -\gamma x_2 + \delta x_1 x_2 \qquad \text{for } \alpha, \beta, \gamma, \delta \in \mathbb{R}^+$$

ii)

$$\dot{x}_1 = x_2 + 5 \qquad \dot{x}_2 = \cos(x_1)$$

iii)

$$\dot{x}_1 = e^{x_1} + \alpha x_2 + 5x_1 - \beta \qquad \dot{x}_2 = x_1^3 \qquad \text{for } \alpha, \beta \in \mathbb{R}$$

iv)

$$\dot{x}_1 = 2x_2 - x_2^2 - 2x_1 x_2 \qquad \dot{x}_2 = 2x_1 - x_1^2 - 2x_1 x_2$$

2) Bring the coefficient matrix for each of the following linear systems into the Jordan form. Determine the nature of their fixed points thereafter.

i)

$$\dot{x}_1 = x_2 \qquad \dot{x}_2 = -2x_1 + 3x_2$$

ii)

$$\dot{x}_1 = -2x_1 + 2x_2 \qquad \dot{x}_2 = -2x_1 + 3x_2$$

iii)

$$\dot{x}_1 = x_1 - x_2 \qquad \dot{x}_2 = x_1 + 3x_2$$

3) Near the fixed points draw the local phase portraits of the following nonlinear systems. Exploit the fact that the isoclines are trajectories which can be drawn easily. Does the linearization theorem apply?

i)

$$\begin{aligned} \dot{x}_1 &= 12x_1 - 3x_1^2 - 4x_1 x_2 \\ \dot{x}_2 &= 4x_2 - x_1^2 - 2x_1 x_2 \ . \end{aligned}$$

ii)

$$\begin{aligned} \dot{x}_1 &= -x_2^7 \\ \dot{x}_2 &= x_1 + x_2^4 \ . \end{aligned}$$