Dynamical Systems Exercises 2

1) Consider the system

$$\begin{aligned} \dot{x}_1 &= 2x_1 - x_1x_2 \\ \dot{x}_2 &= 2x_1x_2 - x_2^2 - 2x_2 \ . \end{aligned}$$

- Find and classify the fixed points, if possible use the linearization theorem.
- Find the principal directions of the trajectories at the fixed points.
- Draw the phase portrait locally at each of the fixed points.
- Indicate where $x_1(t)$ and $x_2(t)$ are increasing and decreasing functions.

2) Consider the system

$$\begin{aligned} \dot{x}_1 &= x_1(3 - x_1 - x_2) \\ \dot{x}_2 &= x_2(6 - 3x_1 - x_2) \ . \end{aligned}$$

- Find and classify the fixed points, if possible use the linearization theorem.
- Find the principal directions of the trajectories at the fixed points.
- Draw the phase portrait locally at each of the fixed points.
- Indicate where $x_1(t)$ and $x_2(t)$ are increasing and decreasing functions.

3) When possible use the linearization theorem to classify the fixed points of the following system

$$\begin{aligned} \dot{x}_1 &= -x_1(2 - x_1^2 - x_2^2) \\ \dot{x}_2 &= -x_2(1 + x_1^2 + x_2^2 - 3x_1) \end{aligned}$$

Using $V(x_1, x_2) = x_1^2 + x_2^2$ as a candidate for a Lyapunov function, show that $x_1^2 + x_2^2 < 1/9$ is a domain of stability.