Dynamical Systems Exercises 3

1) Sketch the phase portraits for each of the following systems and find the corresponding α and ω limit cycles.

i)

$$\begin{aligned} \dot{r} &= r(1-r)(r-2) \\ \dot{\theta} &= 1\ . \end{aligned}$$

ii)

$$\begin{aligned} \dot{r} &= \begin{cases} 0 & \text{for r} \leq 1 \\ r(r-1) & \text{otherwise} \end{cases} \\ \dot{\theta} &= -1\ . \end{aligned}$$

iii)

$$\begin{aligned} \dot{r} &= -r(r-2)^2 \\ \dot{\theta} &= 1\ . \end{aligned}$$

2) Show that the system

$$\begin{aligned} \dot{x}_1 &= x_1(2-x_1^2-x_2^2)-2x_2 \\ \dot{x}_2 &= x_2(2-x_1^2-x_2^2)+3x_1 \end{aligned}$$

has a fixed point at the origin and classify it.

i) Transform the system to polar coordinates and show that the fixed point at the origin is the only one of the system.

ii) Show further that the system has a period orbit in the annular region $1 \leq r \leq 2$.

iii) Determine some values $\mathrm{r}_{\mathrm{min}}$ and $\mathrm{r}_{\mathrm{max}}$, such that the orbit is in the smaller annular region $1 < r_{\mathrm{min}} \leq r \leq r_{\mathrm{max}} < 2$.

3) Prove that none of the following dynamical systems has any limit cycle

i)

$$\dot{x}_1 = x_1 + 3x_2^2 \qquad \text{and} \qquad \dot{x}_2 = -2x_1 - x_2(1+x_1^2+x_1^4)$$

ii)

$$\dot{x}_1 = x_1 - x_1^2 + 2x_2^2 \qquad \text{and} \qquad \dot{x}_2 = x_1x_2 + x_2\ .$$