Geometry & Vectors

Coursework 1

(Hand in the solutions to all questions on Friday 18/02/05 14:00)

1) (10 marks) Consider three distinct planes $\mathcal{P}_1, \mathcal{P}_2$ and $\mathcal{P}_3$, which all intersect with each other. Show that their lines of intersection are either all parallel to each other or cross in one point.

2) (10 marks) The vectors $\vec{\imath}, \vec{\jmath}, \vec{k}$ constitute an orthonormal basis. Given are the two vectors

$$\vec{u} = \kappa\vec{\imath} - 4\vec{\jmath} + 6\vec{k} \qquad \text{and} \qquad \vec{v} = 2\vec{\imath} + \frac{\kappa}{2}\vec{\jmath} + 9\vec{k} \qquad \text{with } \kappa \in \mathbf{R} .$$

Determine the constant κ, such that the angle between the vectors $\vec{u}$ and $\vec{v}$ is $\pi/4$.

3) (25 marks) Two sides of a triangle are formed by the vectors

$$\vec{u} = \frac{1}{2}\left(\tau_+\vec{\imath} - \vec{\jmath} + \tau_-\vec{k}\right) \qquad \text{and} \qquad \vec{v} = \frac{1}{2}\left(-\tau_+\vec{\imath} + \vec{\jmath} + \tau_-\vec{k}\right),$$

where $\tau_\pm = \frac{1}{2}(\sqrt{5} \pm 1)$.

i) Determine all angles in the triangle.

ii) Find a vector unit vector $\vec{w}$ which is perpendicular to $\vec{u}$ and forms an angle $2\pi/3$ with the vector $\vec{v}$.

(Hint: The computations simplify if you make use of the fact that τ_+ is the golden ratio and τ_- its inverse, such that $\tau_\pm^2 = 1 \pm \tau_\pm$.)

4) (5 marks) An object is moved from a point $A(2,1,3)$ to a point $B(4,-1,\mu)$ by a force $\vec{F} = 5\vec{\imath} - 3\vec{\jmath} + 2\vec{k}$. Do to this you do not want to use more than 30 units of work. Determine the point B, that is find the constant μ.

5) (25 marks) ABC constitutes a triangle and D is a point on the line $\overleftrightarrow{AB}$ between the points A and B. Use the dot product to prove the following identity

$$|\overrightarrow{AC}|^2|\overrightarrow{BD}| + |\overrightarrow{BC}|^2|\overrightarrow{AD}| - |\overrightarrow{CD}|^2|\overrightarrow{AB}| = |\overrightarrow{AB}||\overrightarrow{AD}||\overrightarrow{BD}| .$$

6) (25 marks) ABCD constitutes a parallelogram. Attach to each side of the parallelogram externally a square. Sketch the corresponding figure and prove that the centres of the four squares also form a square.