Geometry & Vectors

Coursework 1
(Hand in the solutions to all questions by Tuesday 28/02/06 16:00)

1) (5 marks) What is meant by an axiom, a theorem, a lemma, a corollary and a proposition? Provide a short definition for each of these notions.

2) (15 marks) The vectors $\vec{\imath}, \vec{\jmath}, \vec{k}$ constitute an orthonormal basis. Given are the two arbitrary vectors

$$\vec{u} = -\vec{\imath} + \lambda\vec{\jmath} + \vec{k} \qquad \text{and} \qquad \vec{v} = 2\vec{\jmath} - \vec{k} \qquad \text{with } \lambda \in \mathbb{R} \ .$$

i) Find all vectors which are perpendicular to the plane which contains $\vec{u}$ and $\vec{v}$. Subsequently determine λ such that these vectors have length 3.

ii) Determine the constant λ, such that the angle between the vectors $\vec{u}$ and $\vec{v}$ is $\pi/6$.

iii) For $\lambda = 1$ verify the Cauchy-Schwarz and the triangle inequality

$$-|\vec{u}|\,|\vec{v}| \leq \vec{u} \cdot \vec{v} \leq |\vec{u}|\,|\vec{v}| \qquad \text{and} \qquad |\vec{u} + \vec{v}| \leq |\vec{u}| + |\vec{v}| \, ,$$

respectively. Can one find a λ such that the equal signs holds in these identities?

3) (10 marks) Given are three arbitrary vectors $\vec{u}$, $\vec{v}$ and $\vec{w}$.

i) Prove that

$$\vec{u} \times \vec{v} \cdot \vec{u} = 0$$

ii) Use the identity in i) to simplify the expression

$$(\vec{u} + \vec{v}) \cdot (\vec{v} + \vec{w}) \times (\vec{w} + \vec{u})$$

4) (10 marks) By evaluating the cross product between two unit vectors prove the two identities

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta.$$

5) (10 marks) ABC constitutes a triangle. The point D is on the line $\overleftrightarrow{BC}$ between the points B and C, the ponit E is on the line $\overleftrightarrow{AC}$ between the points A and C and the point F is on the line $\overleftrightarrow{AB}$ between the points A and B. The following ratios hold

$$\frac{BD}{BC} = \frac{CE}{CA} = \frac{AF}{AB} = \frac{2}{3}.$$

Sketch the corresponding figure and show that

$$\overrightarrow{AD} = \overrightarrow{EB} + \overrightarrow{FC}.$$