MA1607

CITY UNIVERSITY

London

BSc Degrees in Mathematical Science
Mathematical Science with Statistics
Mathematical Science with Computer Science
Mathematical Science with Finance and Economics
MMath Degrees in Mathematical Science

PART I EXAMINATION

Geometry and Vectors

May 2008

Time allowed: 2 hours

Full marks may be obtained for correct answers to FOUR questions of section A and TWO questions of section B.

The best marks will be credited if more questions are answered.

Section **A**

Answer **four** questions from this section. Each question carries 12 marks.

1. The vectors $\vec{\imath}, \vec{\jmath}, \vec{k}$ constitute an orthonormal basis in an Euclidean space. The vectors $\vec{u}$ and $\vec{v}$ are defined by

$$\vec{u} = \lambda\vec{\imath} - 7\vec{\jmath} - \vec{k}, \qquad \text{and} \qquad \vec{v} = 2\vec{\imath} - \vec{\jmath} + 2\vec{k} \qquad \text{with } \lambda \in \mathbb{R}.$$

(i) Determine the constant λ such that the angle between $\vec{u}$ and $\vec{v}$ becomes $\pi/4$.

(ii) Take now $\lambda = -1$ and construct all vectors with length $\sqrt{90}$ which are perpendicular to both vectors $\vec{u}$ and $\vec{v}$.

(iii) Compute the expression $\vec{u} \times \vec{v}$ for $\lambda = 14$.

2. The vectors $\vec{a}, \vec{b}, \vec{c}, \vec{x}$ are arbitrary and $\lambda \in \mathbb{R}$ is a scalar.

(i) For given vectors $\vec{a}, \vec{b}$ and $\vec{c}$ find the general expression for the vector $\vec{x}$, which solves the vector equation

$$\lambda\vec{x} + (\vec{x} \cdot \vec{b})\vec{a} = \vec{c} \qquad \lambda \neq 0.$$

[Hint: Treat the cases $\lambda + \vec{a} \cdot \vec{b} \neq 0$ and $\lambda + \vec{a} \cdot \vec{b} = 0$ separately.]

(ii) Use the result from (i) to solve the vector equation

$$\vec{x} \times \vec{a} = \vec{b}$$

for $\vec{x}$ when $\vec{a}$ and $\vec{b}$ are given.

[Hint: You may use the identity $\vec{u} \times \vec{v} \times \vec{w} = (\vec{u} \cdot \vec{w})\vec{v} - (\vec{u} \cdot \vec{v})\vec{w}$.]

3. A circle with radius 2 and center located on the y-axis is inscribed into the parabola $y = x^2/2$. (This means the circle and the parabola have the same tangent at the points of intersection.)

(i) Draw the corresponding figure.

(ii) Determine the points of intersection, the center of the circle and the intersection of the circle with the y-axis.

Turn over ...

4. Given the two points $A(6, 1, 3)$ and $B(4, 5, 1)$,

(i) find the equation of the line passing through A and B by determining the point of intersection of this line with the yz-plane;

(ii) find the coordinates of the point in which the line through the points A and B intersects the plane

$$\mathcal{P}: \quad 2x + y - 3z = 16.$$

5. $ABCD$ constitutes a parallelogram. The point W is the midpoint of the line segment BC. The lines $\overleftrightarrow{AW}$ and $\overleftrightarrow{BD}$ intersect in the point X.

(i) Sketch the corresponding figure.

(ii) State the similarity axiom.

(iii) Use the similarity axiom to show that

$$DX : XB = 2 : 1.$$

Turn over ...

Section **B**

Answer **two** questions from this section. Each question carries 26 marks.

6. Given the three position vectors $\vec{a} = \overrightarrow{OA}$, $\vec{b} = \overrightarrow{OB}$, $\vec{c} = \overrightarrow{OC}$ and a point D situated on the line $\overleftrightarrow{BC}$, with $\vec{d}$ denoting the vector $\overrightarrow{AD}$;

(i) draw the corresponding figure;

(ii) use vectors to show that the shortest distance from the point A to the point D is given by the expression

$$|\vec{d}| = \frac{|\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}|}{|\vec{b} - \vec{c}|};$$

(iii) take now the vectors

$$\vec{a} = -\frac{1}{4}\vec{\imath} + \vec{\jmath}, \qquad \vec{b} = \vec{\imath}, \qquad \vec{c} = \frac{5}{4}\vec{\imath} + \frac{3}{2}\vec{\jmath}$$

and compute $|\vec{d}|$;

(iv) compute the position vector $\overrightarrow{OD}$.

7. Given are the three points $A(0,3,1)$, $B(2,4,0)$ and $C(3,5,5)$ and the two lines

$$\mathcal{L}_1 \;:\; \frac{x+1}{2} = y - 1 = \frac{z-2}{3}$$
$$\mathcal{L}_2 \;:\; -x = \frac{y+9}{3} = z + 4.$$

(i) Do the two lines $\mathcal{L}_1$ and $\mathcal{L}_2$ intersect? In case they do, find the coordinates of their point of intersection $P = \mathcal{L}_1 \cap \mathcal{L}_2$.

(ii) Determine the equation of the plane $\mathcal{P}_1$ containing $\mathcal{L}_1$ and $\mathcal{L}_2$, i.e. $\mathcal{L}_1 \in \mathcal{P}$, $\mathcal{L}_2 \in \mathcal{P}$.

(iii) Determine the equation of the plane $\mathcal{P}_2$ which contains the points A, B and C.

(iv) Compute the equation for the line of intersection for the two planes $\mathcal{P}_1$ and $\mathcal{P}_2$ in Cartesian form.

Turn over ...

8. Given the two spheres

$$\mathcal{S}_1 \quad : \quad x^2 + y^2 + z^2 = 9$$
$$\mathcal{S}_2 \quad : \quad x^2 + (y+6)^2 + z^2 = 36$$

and the plane

$$\mathcal{P}: \; \lambda x + \mu y + z = 6,$$

(i) show that the condition for $\mathcal{P}$ to be a tangent plane to $\mathcal{S}_1$ is

$$\lambda^2 + \mu^2 = 3;$$

(ii) find the condition for $\mathcal{P}$ to be also a tangent plane to $\mathcal{S}_2$;

(iii) determine the equation of the plane which is a common tangent plane to $\mathcal{S}_1$ and $\mathcal{S}_2$;

(iv) specify a vector which lies in the plane determined in (iii).

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