LOGARITHMIC SUPERCONFORMAL FIELD THEORY

[illegible]

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- MOTIVATION + DEFINITIONS
- BOSONIC CASE
  - PRIMARY FIELDS
  - 2 PT FUNCTION
- $N = 1$
- $N = 2$
- CONCLUSIONS

WHERE CAN WE SEE LOGARITHMIC CFT IN ACTION?

- STATISTICAL PHYSICS

  ABELIAN SANDPILE MODEL

- WZW THEORIES

  $SU(2)_0$ OSP[illegible]

- D-BRANE DYNAMICS

  [illegible]

WHAT IS LOGARITHMIC CFT?

- PROPER DEFINITION ...?

- NON-UNITARY CFTs,
  LOGARITHMIC DIVERGENCES IN
  CORRELATORS

  CARTAN SUB-ALGEBRA ELEMENTS
  NON-DIAGANOLIZABLE

HOW TO BUILD THEM?

- OPERATORS OF WEIGHT ≤ 0

  ENHANCED ZERO MODE STRUCTURE

- INTRODUCE JORDAN BLOCKS

$$L_0 \sim \left(\begin{array}{c|ccc} h_1 & & & \\ \hline & h_2 & 0 & \\ & 0 & h_2 & \\ & & & \ddots \end{array}\right)$$

$$\downarrow$$

$$L_0 \sim \left(\begin{array}{c|ccc} h_1 & & & \\ \hline & h_2 & 1 & \\ & 0 & h_2 & \\ & & & \ddots \end{array}\right)$$

h NORMALLY ENCODED BY A PRIMARY FIELD – SECTION OF A LINE BUNDLE dz^h – WHAT ARE PRIMARY FIELDS NOW?

BOSONIC CASE - PRIMARY FIELDS

RANK 2 $\quad L_0 \begin{pmatrix} |\phi_0\rangle \\ |\phi_1\rangle \end{pmatrix} = \begin{pmatrix} h & 1 \\ 0 & h \end{pmatrix} \begin{pmatrix} |\phi_0\rangle \\ |\phi_1\rangle \end{pmatrix}$

INSTEAD OF $\quad L_0 |\phi_i\rangle = h|\phi_i\rangle$

CONSIDER, FORMALLY

$$dz^{h\mathbb{1} + J}$$

WHERE $h\mathbb{1} + J = \begin{pmatrix} h & 1 \\ 0 & h \end{pmatrix}$, $\quad a^b = \exp(b \log a)$

$$\Rightarrow \quad dz^{h\mathbb{1}+J} = dz^h (1 + J \log(dz))$$

USUALLY dz^h GENERATES TRANSITION FUNCTIONS ON A LINE BUNDLE - NOW SEEMS TO GIVE A MATRIX.

COULD LET MATRIX ACT ON A VECTOR OR ANOTHER MATRIX

$$\begin{pmatrix} \phi_0(z) \\ \phi_1(z) \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} \phi_1(z) & \phi_0(z) \\ 0 & \phi_1(z) \end{pmatrix} = \underline{\phi}(z)$$

$$= \phi_1 + J\phi_0$$

CONSIDER TRANSFORMATION

$$z \longmapsto z' = f(z)$$

$$\Rightarrow \left(\begin{array}{c|c} \phi_1'(z) & \phi_0'(z) \\ \hline 0 & \phi_1'(z) \end{array}\right) = \left(\frac{dz'}{dz}\right)^h \left(\begin{array}{c|c} \phi_1(z') & \phi_0(z') + \log\left(\frac{dz'}{dz}\right)\phi_1(z') \\ \hline 0 & \phi_1(z') \end{array}\right)$$

i.e. $$\underline{\phi}'(z) = \left(\frac{dz'}{dz}\right)^{h+J} \underline{\phi}(z')$$

INFINITESIMAL TRANSFORMATIONS

$$f(z) = z + a z^{n+1}$$

$$\Rightarrow \delta_{az^{n+1}} \underline{\phi} = a z^n \left((h+J)(n+1) + z \frac{d}{dz} \right) \underline{\phi}(z)$$

$$= a \left[L_n , \underline{\phi}(z) \right]$$

SIMILARLY FOR RANK $N > 2$

TAKE $J^N = 0$, $J^{N-1} \neq 0$

AND $$\underline{\phi}(z) = \sum_{i=0}^{N-1} \phi_i(z) J^{N-1-i}$$

BOSONIC CASE - TWO PT FUNCTION

NORMALLY $\{L_{-1}, L_0, L_1\} \Rightarrow$

$$\langle \phi(z)\psi(w)\rangle = f(z,w) = \frac{a}{(z-w)^{h_1+h_2}}$$

$$(h_1-h_2) \neq 0 \Rightarrow a = 0$$

NOW

$$\langle \underline{\phi}(z,J) \otimes \underline{\psi}(w,K)\rangle = f(z,w,J,K)$$

$$= \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} f_{mn}(z,w)\, J^m \otimes K^n$$

WHERE $J^M = 0$, $J^{M-1} \neq 0$

$K^N = 0$, $K^{N-1} \neq 0$

$$\underline{\phi}(z,J) = \sum_{i=0}^{M-1} \phi_i(z)\, J^{M-1-i}$$

$$\underline{\psi}(w,K) = \sum_{i=0}^{N-1} \psi_i(z)\, K^{N-1-i}$$

GET

$$\langle \underline{\Phi}(z,J) \otimes \underline{\Psi}(\omega,K) \rangle = \frac{C(J,K)}{(z-\omega)^{h_1+h_2+J+K}}$$

$$h_1 - h_2 \neq 0 \Rightarrow C = 0$$

$$(J-K)C \neq 0 \Rightarrow C = 0$$

E.G. $M = N = 2$

$$\langle \underline{\Phi} \; \underline{\Psi} \rangle = (z-\omega)^{-2h}\Big((J+K)a + JK\big(b - 2a\log(z-\omega)\big)\Big)$$

$M = 2$, $N = 3$

$$\langle \underline{\Phi} \; \underline{\Psi} \rangle = (z-\omega)^{-2h}\Big((K^2+JK)a + JK^2\big(b - 2a\log(z-\omega)\big)\Big)$$

3-PT FN , $\{L_{-1}, L_0, L_1\}$

$$\Rightarrow \langle \Phi_1(x, J)\, \Phi_2(y, K)\, \Phi_3(z, L) \rangle$$

$$= C(J,K,L)\,(x-y)^{-h_1-h_2+h_3-J-K+L} \times$$
$$(y-z)^{-h_2-h_3+h_1-K-L+J} \times$$
$$(x-z)^{-h_1-h_3+h_2-J-K+L}$$

$N=1$ ON (z, θ) SUPERSPACE

NORMALLY $N=1$ PRIMARY FIELDS

SECTIONS OF ω^h, $\omega = dz + \theta d\theta$

PLAY THE SAME GAME ... $h \to h+J$ ...

$$[L_n, \underline{\phi}(z,\theta)]$$

$$= z^n\left((h+J)(n+1) + z\frac{\partial}{\partial z} + \frac{1}{2}(n+1)\,\theta\frac{\partial}{\partial \theta} \right) \underline{\phi}(z,\theta)$$

AND SUPERSYMMETRIC PARTNER

$$[G_r, \underline{\phi}(z,\theta)]$$

$$= z^{r-1/2}\left((h+J)(r+\tfrac{1}{2})\theta + z\theta\frac{\partial}{\partial z} - z\frac{\partial}{\partial \theta} \right) \underline{\phi}(z,\theta)$$

. . .

$N=2$ ON (z, θ^+, θ^-) SUPERSPACE

CARTAN SUBALGEBRA NO LONGER

$$\{L_0, C\}, \quad \text{BUT} \quad \{L_0, \underset{\substack{\uparrow \\ \text{R-SYMMETRY}}}{T_0}, C\}$$

PRIMARY FIELDS SECTIONS OF

$$\omega^{h - q/2} \otimes D_+^{\,q}$$

$\omega = dz + \theta^+ d\theta^- + \theta^- d\theta^+$

$D_+ = \frac{\partial}{\partial \theta^+} + \theta^- \frac{\partial}{\partial z}$

i.e. $\phi'(z, \theta^+, \theta^-)$

$$= (D_+ \theta^{+\prime})^{h + \frac{q}{2}} (D_- \theta^{-\prime})^{h - \frac{q}{2}} \phi(z', \theta^{+\prime}, \theta^{-\prime})$$

CAN NOW REPLACE

$$h \to h + J$$

$$q \to q + P$$

TWO PT FUNCTION

$$f = \langle \phi(z, \theta_i^+, J, P) \otimes \psi(w, \chi_i^+, K, Q) \rangle$$

$$h_1 \to h_1 + J \qquad\qquad h_2 \to h_2 + K$$

$$q_1 \to q_1 + P \qquad\qquad q_2 \to q_2 + Q$$

$$\{ L_{-1}, L_0, L_1, G^{\pm}_{1/2}, G^{\pm}_{-1/2}, T_0 \}$$

SUSY & SPECIAL SUSY

R-SYMMETRY

$$\Rightarrow \quad f = C(J, K, P, Q)\left(Z_{12}^{-\Delta} - (q_2 + Q)\,\theta^{+}_{12}\theta^{-}_{12} Z_{12}^{-\Delta-1} \right)$$

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f non-zero $\Rightarrow$

$$h_1 - h_2 = 0$$
$$q_1 + q_2 = 0$$
$$(J-K)C = 0$$
$$(P+Q)C = 0$$

P & Q DO NOT GENERATE LOGARITHMS!

CONCLUSIONS

- FOUND A SUITABLE FORMALISM FOR LOG PRIMARY FIELDS THAT GENERALIZES TO SUSY
- 'CENTRAL CHARGE' ?
- 3-PT FNS AND OPERATOR ALGEBRA ?
- UNDERLYING GEOMETRY ?