CITY UNIVERSITY LONDON

Name:

Classtest Mathematical Methods II, MA3603 (A)

Instructions

Answer all five questions by clearly marking the box of the correct answer. Each question carries 6 marks. Some of the questions may have several correct answers, in which case the 6 marks are distributed equally over the correct answers. A wrong answer will annihilate the marks of a correct answer. Remove any notes from your workplace.

DATE: Monday 08/04/2013 at 13:00

1) Consider the following transformations.

$$T_1(z) = \frac{z + 2e^{i\pi/6}}{2z + i\sqrt{3}}, \quad T_2(z) = \frac{z^2 + iz + z\sqrt{3}}{2z^2 + zi\sqrt{3}},$$
$$T_3(z) = \frac{z + i\left(z + \sqrt{3} + 1\right) + \sqrt{3} - 1}{(2+2i)z - (1-i)\sqrt{3}}, \quad T_4(z) = \frac{z - i - \sqrt{3}}{2z + i\sqrt{3}},$$

Which of the following statements is correct?

☐ All transformations are inequivalent.

■ $T_1(z)$ and $T_2(z)$ are equivalent.

■ $T_2(z)$ and $T_3(z)$ are equivalent.

☐ $T_1(z)$ and $T_4(z)$ are equivalent.

■ $T_1(z)$ and $T_3(z)$ are equivalent.

☐ $T_3(z)$ and $T_4(z)$ are equivalent.

2) Given is the function $f(z) = x^3 - 3xy^2 + i(3x^2y - y^3)$ defined on $D \in \mathbb{C}$. Which of the following statements is correct?

☐ The derivative of $f(z)$ is not an analytic function.

☐ When $f'(z_0) \neq const$ then $f(z)$ preserves angles at $z_0 \in D$.

■ $u(x,y) = x^3 - 3xy^2$ is a harmonic functions.

■ $3x^2y - y^3$ is the conjugate function of $x^3 - 3xy^2$.

■ $f(z)$ is a conformal map.

☐ D is conformal.

3) Which of the following statements is correct?

☐ Branch cuts make a multivalued function single valued and analytic.

■ Using Riemann surfaces one does not need branch cuts to make functions analytic.

■ $f(r,\theta) = r^{1/4}e^{i\theta/4}$ and $g(r,\theta) = r^{2/8}e^{i\theta/4}e^{i3\pi/4}$ are branches of the same function.

☐ Every point on a branch cut is a branch point.

4) Which of the following identities is correct

$$I_1 : \int_{-1}^{-\infty} d\hat{z}(\hat{z}-1)^{-2/3}(\hat{z}+1)^{-2/3} = e^{-i\pi/3}\int_1^{\infty} d\hat{z}\,|\hat{z}+1|^{-2/3}\,|\hat{z}-1|^{-2/3}$$

$$I_2 : \int_{-1}^{-\infty} d\hat{z}(\hat{z}-1)^{-2/3}(\hat{z}+1)^{-2/3} = e^{-i\pi 4/3}\int_1^{\infty} d\hat{z}\,|\hat{z}+1|^{-2/3}\,|1-\hat{z}|^{-2/3}$$

$$I_3 : 1 = -\int_{-\infty}^{\infty} \delta(x-\pi)\sin(3x/2)dx$$

$$I_4 : \mathcal{F}u'(x) = ix\mathcal{F}u(x) \text{ when } \lim_{t\to\pm\infty} u(t) = 1$$

$$I_5 : \mathcal{F}v(x) = e^{x\Delta}\mathcal{F}u(x) \qquad \text{for } v(x) = u(x+\Delta),\ \Delta\in\mathbb{R}$$

☐ All identities are incorrect.

■ I_1 is correct.

☐ I_2 is correct.

■ I_3 is correct.

☐ I_4 is correct.

☐ I_5 is correct.

5) The function $u(x)$ is piecewise smooth and has exponential growth $\alpha > 0$. The function $v(x)$ is piecewise smooth and has exponential growth $\beta > \alpha$. The function $w(x)$ is absolutely integrable.

Which of the following statements is correct?

☐ The Laplace transform for the function $w(x)$ exists.

☐ $\mathcal{L}(u \star v)(x) = (\mathcal{L}u)(x)(\mathcal{L}v)(x)$ for $x > \alpha$.

■ $\mathcal{L}(u \star v)(x) = (\mathcal{L}u)(x)(\mathcal{L}v)(x)$ for $x > \beta$.

☐ $\mathcal{L}(u \star v)(x) = (\mathcal{L}u)(x)(\mathcal{L}v)(x)$ for $\alpha < x < \beta$.

☐ $\mathcal{L}(u \star v)(x) = (\mathcal{L}u)(x)(\mathcal{L}v)(x)$ for $x > 0$.

☐ $\mathcal{L}(\lambda u + \kappa v)(x) = \bar{\lambda}\mathcal{L}(u)(x) + \bar{\kappa}\mathcal{L}(v)(x), \quad \lambda, \kappa \in \mathbb{C}$.

CITY UNIVERSITY LONDON

Name:

Classtest Mathematical Methods II, MA3603 (B)

Instructions

Answer all five questions by clearly marking the box of the correct answer. Each question carries 6 marks. Some of the questions may have several correct answers, in which case the 6 marks are distributed equally over the correct answers. A wrong answer will annihilate the marks of a correct answer. Remove any notes from your workplace.

DATE: Monday 08/04/2013 at 13:00

1) Consider the following transformations.

$$T_1(z) = \frac{z+\sqrt{2}e^{i\pi/4}}{3z+i\sqrt{2}}, \quad T_2(z) = \frac{z-1-i}{3z+i\sqrt{2}},$$
$$T_3(z) = \frac{z^2+(1+i)z}{3z^2+i\sqrt{2}z}, \quad T_4(z) = \frac{2+(1-i)z}{(3-3i)z+(1+i)\sqrt{2}},$$

Which of the following statements is correct?

☐ All transformations are inequivalent.

☐ $T_1(z)$ and $T_2(z)$ are equivalent.

☐ $T_2(z)$ and $T_3(z)$ are equivalent.

■ $T_1(z)$ and $T_4(z)$ are equivalent.

■ $T_1(z)$ and $T_3(z)$ are equivalent.

■ $T_3(z)$ and $T_4(z)$ are equivalent.

2) Given is the function $f(z) = x^3 - 3xy^2 + i(3x^2y - y^3)$ defined on $D \in \mathbb{C}$. Which of the following statements is correct?

■ $f(z)$ can be written as $f(z) = z^3$.

■ $u(x, y) = 3x^2y - y^3$ is a harmonic functions.

☐ $3x^2y - y^3$ is the conjugate function of $3y^2x - x^3$.

☐ The derivative of $f(z)$ is not an analytic function.

☐ D is conformal.

■ $f(z)$ is a conformal map.

3) Which of the following statements is correct?

☐ Every point on a branch cut is a branch point.

■ To avoid branch cuts to make functions analytic one can use Riemann surfaces .

■ $f(r,\theta) = r^{1/3}e^{i\theta/3}e^{i\pi/3}$ and $g(r,\theta) = r^{2/6}e^{i\theta/3}$ are branches of the same function.

☐ Branch cuts make a multivalued function single valued and analytic.

4) Which of the following identities is correct

$$I_1 : \int_{-1}^{-\infty} d\hat{z}(\hat{z}-1)^{-2/3}(\hat{z}+1)^{-2/3} = e^{-i\pi 4/3} \int_1^{\infty} d\hat{z}\,|\hat{z}+1|^{-2/3}\,|\hat{z}-1|^{-2/3}$$

$$I_2 : \int_{-1}^{-\infty} d\hat{z}(\hat{z}-1)^{-2/3}(\hat{z}+1)^{-2/3} = e^{-i\pi/3} \int_1^{\infty} d\hat{z}\,|\hat{z}+1|^{-2/3}\,|1-\hat{z}|^{-2/3}$$

$$I_3 : \mathcal{F}v(x) = e^{x\Delta}\mathcal{F}u(x) \qquad \text{for } v(x) = u(x+\Delta),\ \Delta \in \mathbb{R}$$

$$I_4 : \mathcal{F}u'(x) = x\mathcal{F}u(x) \ \text{ when } \lim_{t\to\pm\infty} u(t) = 0$$

$$I_5 : -1 = \int_{-\infty}^{\infty} \delta(x-\pi)\cos(3x)dx$$

☐ All identities are incorrect.

☐ I_1 is correct.

■ I_2 is correct.

☐ I_3 is correct.

☐ I_4 is correct.

■ I_5 is correct.

5) The function $u(x)$ is piecewise smooth and has exponential growth $\gamma > 0$. The function $v(x)$ is piecewise smooth and has exponential growth $\beta > \gamma$. The function $w(x)$ is absolutely integrable.

Which of the following statements is correct?

☐ The Laplace transform for the function $w(x)$ exists.

☐ $\mathcal{L}(\lambda u + \kappa v)(x) = \kappa\mathcal{L}(u)(x) + \lambda\mathcal{L}(v)(x), \qquad \lambda, \kappa \in \mathbb{C}$.

☐ $\mathcal{L}(u \star v)(x) = (\mathcal{L}u)(x)(\mathcal{L}v)(x)$ for $x > 0$.

☐ $\mathcal{L}(u \star v)(x) = (\mathcal{L}u)(x)(\mathcal{L}v)(x)$ for $\gamma < x < \beta$.

☐ $\mathcal{L}(u \star v)(x) = (\mathcal{L}u)(x)(\mathcal{L}v)(x)$ for $x > \gamma$.

■ $\mathcal{L}(u \star v)(x) = (\mathcal{L}u)(x)(\mathcal{L}v)(x)$ for $x > \beta$.

CITY UNIVERSITY LONDON

Name:

Classtest Mathematical Methods II, MA3603 (C)

Instructions

Answer all five questions by clearly marking the box of the correct answer. Each question carries 6 marks. Some of the questions may have several correct answers, in which case the 6 marks are distributed equally over the correct answers. A wrong answer will annihilate the marks of a correct answer. Remove any notes from your workplace.

DATE: Monday 08/04/2013 at 13:00

1) Consider the following transformations.

$$T_1(z) = \frac{z-1-i\sqrt{3}}{\sqrt{3}z+i}, \quad T_2(z) = \frac{z+2e^{i\pi/3}}{\sqrt{3}z+i},$$
$$T_3(z) = \frac{z+4-i\sqrt{3}z}{i(1-3z)+\sqrt{3}z+\sqrt{3}}, \quad T_4(z) = \frac{-z^2-i\sqrt{3}z-z}{-\sqrt{3}z^2-iz},$$

Which of the following statements is correct?

☐ All transformations are inequivalent.

☐ $T_1(z)$ and $T_2(z)$ are equivalent.

■ $T_2(z)$ and $T_3(z)$ are equivalent.

☐ $T_1(z)$ and $T_4(z)$ are equivalent.

■ $T_2(z)$ and $T_3(z)$ are equivalent.

■ $T_3(z)$ and $T_4(z)$ are equivalent.

2) Given is the function $f(z) = x^3 - 3xy^2 + i(3x^2y - y^3)$ defined on $D \in \mathbb{C}$. Which of the following statements is correct?

■ $f(z)$ preserves angles of intersection lines.

☐ $u(x,y) = 3x^2y - y^2$ is a harmonic functions.

■ $3x^2y - y^3$ is the conjugate function of $x^3 - 3xy^2$.

■ The derivative of $f(z)$ is an analytic function.

☐ D is not conformal.

☐ $f(z)$ is not a conformal map.

3) Which of the following statements is correct?

■ $f(r,\theta) = r^{1/5}e^{i\theta/5}e^{i\pi/5}$ and $g(r,\theta) = r^{2/10}e^{i\theta/5}$ are branches of the same function.

■ Use Riemann surfaces instead of branch cuts avoids to restrict θ.

☐ Every point on a branch cut is a branch point.

☐ Branch cuts make a multivalued function single valued and analytic.

4) Which of the following identities is correct

$$I_1 : \int_{-1}^{-\infty} d\hat{z}(\hat{z}-1)^{-2/3}(\hat{z}+1)^{-2/3} = e^{-i\pi/3}\int_1^{\infty} d\hat{z}\,|\hat{z}+1|^{-2/3}\,|\hat{z}-1|^{-2/3}$$
$$I_2 : \mathcal{F}u'(x) = ix\mathcal{F}u(x) \;\; \text{when} \lim_{t\to\pm\infty} u(t) = 0$$
$$I_3 : \mathcal{F}v(x) = e^{x\Delta}\mathcal{F}u(x) \qquad \text{for } v(x) = u(x+\Delta),\ \Delta\in\mathbb{R}$$
$$I_4 : 1 = \int_{-\infty}^{\infty} \delta(x-\pi)\cos(3x)dx$$
$$I_5 : \int_{-1}^{-\infty} d\hat{z}(\hat{z}-1)^{-2/3}(\hat{z}+1)^{-2/3} = e^{-i\pi 4/3}\int_1^{\infty} d\hat{z}\,|\hat{z}+1|^{-2/3}\,|1-\hat{z}|^{-2/3}$$

☐ All identities are incorrect.

■ I_1 is correct.

■ I_2 is correct.

☐ I_3 is correct.

☐ I_4 is correct.

☐ I_5 is correct.

5) The function $u(x)$ is piecewise smooth and has exponential growth $\gamma > 0$. The function $v(x)$ is piecewise smooth and has exponential growth $\alpha > \gamma$. The function $w(x)$ is absolutely integrable.

Which of the following statements is correct?

☐ The Laplace transform for the function $w(x)$ exists.

☐ $\mathcal{L}(\lambda u + \kappa v)(x) = \lambda\mathcal{L}(u)(x) \star \kappa\mathcal{L}(v)(x), \quad \lambda,\kappa\in\mathbb{C}.$

☐ $\mathcal{L}(u \star v)(x) = (\mathcal{L}u)(x)(\mathcal{L}v)(x)$ for $x > 0$.

☐ $\mathcal{L}(u \star v)(x) = (\mathcal{L}u)(x)(\mathcal{L}v)(x)$ for $\gamma < x < \alpha$.

☐ $\mathcal{L}(u \star v)(x) = (\mathcal{L}u)(x)(\mathcal{L}v)(x)$ for $x > \gamma$.

■ $\mathcal{L}(u \star v)(x) = (\mathcal{L}u)(x)(\mathcal{L}v)(x)$ for $x > \alpha$.

CITY UNIVERSITY LONDON

Name:

Classtest Mathematical Methods II, MA3603 (D)

Instructions

Answer all five questions by clearly marking the box of the correct answer. Each question carries 6 marks. Some of the questions may have several correct answers, in which case the 6 marks are distributed equally over the correct answers. A wrong answer will annihilate the marks of a correct answer. Remove any notes from your workplace.

DATE: Monday 08/04/2013 at 13:00

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1) Consider the following transformations.

$$T_1(z) = \frac{2z^2 - i\sqrt{3}z + z}{z^2 + i\sqrt{2}z}, \quad T_2(z) = \frac{2z + 2e^{-i\pi/3}}{z + i\sqrt{2}},$$
$$T_3(z) = \frac{2z + 1 + i\sqrt{3}}{z + i\sqrt{2}}, \quad T_4(z) = \frac{2i\sqrt{3}z + 2z + 4}{z + i\left(\sqrt{3}z + \sqrt{2}\right) - \sqrt{6}},$$

Which of the following statements is correct?

□ All transformations are inequivalent.

■ $T_1(z)$ and $T_2(z)$ are equivalent.

□ $T_2(z)$ and $T_3(z)$ are equivalent.

■ $T_1(z)$ and $T_4(z)$ are equivalent.

□ $T_2(z)$ and $T_3(z)$ are equivalent.

■ $T_2(z)$ and $T_4(z)$ are equivalent.

2) Given is the function $f(z) = x^3 - 3xy^2 + i(3x^2y - y^3)$ defined on $D \in \mathbb{C}$. Which of the following statements is correct?

■ $f(z)$ preserves angles of intersection lines.

■ $f(z)$ is a conformal map.

■ The derivative of $f(z)$ is an analytic function.

□ D is conformal.

□ $u(x, y) = x^3 - 3xy^2$ is not a harmonic functions.

□ $x^3 - 3xy^2$ is the conjugate function of $y^3 - 3yx^2$.

Name: ____

3) Which of the following statements is correct?

☐ Riemann surfaces have infinitely many branch points.

☐ Every point on a branch cut is a branch point.

■ Branch cuts make a nonanalytic functions analytic.

■ $f(r,\theta) = r^{1/7}e^{i\theta/7}e^{i\pi/7}$ and $g(r,\theta) = r^{2/14}e^{i\theta/7}$ are branches of the same function.

4) Which of the following identities is correct

$$I_1 : \int_{-1}^{-\infty} d\hat{z}(\hat{z}-1)^{-2/3}(\hat{z}+1)^{-2/3} = e^{-i\pi 4/3}\int_1^{\infty} d\hat{z}\,|\hat{z}+1|^{-2/3}\,|\hat{z}-1|^{-2/3}$$

$$I_2 : 1 = \int_{-\infty}^{\infty} \delta(x-\pi)\sin(2x)dx$$

$$I_3 : \mathcal{F}u'(x) = ix\mathcal{F}u(x) \quad \text{when} \lim_{t\to\pm\infty} u(t) = \infty$$

$$I_4 : \mathcal{F}v(x) = e^{ix\Delta}\mathcal{F}u(x) \qquad \text{for } v(x) = u(x+\Delta),\ \Delta\in\mathbb{R}$$

$$I_5 : \int_{-1}^{-\infty} d\hat{z}(\hat{z}-1)^{-2/3}(\hat{z}+1)^{-2/3} = e^{-i\pi/3}\int_1^{\infty} d\hat{z}\,|\hat{z}+1|^{-2/3}\,|1-\hat{z}|^{-2/3}$$

☐ All identities are incorrect.

☐ I_1 is correct.

☐ I_2 is correct.

☐ I_3 is correct.

■ I_4 is correct.

■ I_5 is correct.

5) The function $u(x)$ is piecewise smooth and has exponential growth $\beta > 0$. The function $v(x)$ is piecewise smooth and has exponential growth $\alpha > \beta$. The function $w(x)$ is absolutely integrable.

Which of the following statements is correct?

☐ The Laplace transform for the function $w(x)$ exists.

☐ $\mathcal{L}(\lambda u + \kappa v)(x) = -\bar{\lambda}\mathcal{L}(u)(x) - \bar{\kappa}\mathcal{L}(v)(x), \quad \lambda,\kappa\in\mathbb{C}$.

■ $\mathcal{L}(u\star v)(x) = (\mathcal{L}u)(x)(\mathcal{L}v)(x)$ for $x > \alpha$.

☐ $\mathcal{L}(u\star v)(x) = (\mathcal{L}u)(x)(\mathcal{L}v)(x)$ for $x > 0$.

☐ $\mathcal{L}(u\star v)(x) = (\mathcal{L}u)(x)(\mathcal{L}v)(x)$ for $\beta < x < \alpha$.

☐ $\mathcal{L}(u\star v)(x) = (\mathcal{L}u)(x)(\mathcal{L}v)(x)$ for $x > \beta$.